Res No: AB/II(18-19).2.RUA13 / AC/II(18-19).2.RUS8

S.P.Mandali's
RAMNARAIN RUIA AUTONOMOUS COLLEGE, MUMBAI-19

SYLLABUS FOR T.Y.B.Sc /T.Y.B.A

PROGRAM: B.Sc / B.A

COURSE: MATHEMATICS (RUSMAT/RUAMAT)

(Credit Based Semester and Grading System with effect from the academic year 2019–2020)

Semester V

Course Code	Unit	Topics	Credits	L/Week
Integral Calculus				
RUSMAT501/ RUAMAT501	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Algebra II				
RUSMAT502/ RUAMAT502	I	Group Theory	2.5	3
	II	Normal Subgroups		
	III	Direct Products of Groups		
Topology of Metric Spaces				
RUSMAT503/ RUAMAT503	I	Metric Spaces	2.5	3
	II	Closed Sets, Sequences and Completeness		
	III	Continuity		
Graph Theory (Elective I)				
RUSMATE504I/ RUAMATE504I	I	Basics of Graphs	2.5	3
	II	Trees		
	III	Eulerian and Hamiltonian graphs		
Number Theory and its Applications (Elective II)				
RUSMATE504II/ RUAMATE504II	I	Congruences and Factorization	2.5	3
	II	Diophantine Equations and their Solutions		
	III	Primitive Roots and Cryptography		
Course		**Practicals**	**Credits**	**L/Week**
RUSMATP501/ RUAMATP501		Practicals based on RUSMAT501/RUAMAT501 and RUSMAT502/RUAMAT502	3	6
RUSMATP502/ RUAMATP502		Practicals based on RUSMAT503/RUAMAT503, RUSMTE504I/RUAMTE504I or RUSMTE504II/RUAMTE504II	3	6

Semester VI

Course Code	Unit	Topics	Credits	L/Week
Basic Complex Analysis				
RUSMAT601/ RUAMAT601	I	Complex Numbers and Complex functions	2.5	3
	II	Holomorphic functions		
	III	Complex power series		
Algebra III				
RUSMAT602/ RUAMAT602	I	Ring Theory	2.5	3
	II	Factorization		
	III	Field Theory		
Metric Topology				
RUSMAT603/ RUAMAT603	I	Compact sets	2.5	3
	II	Connected sets		
	III	Sequences and Series of functions		
Graph Theory and Combinatorics (Elective I)				
RUSMAT604I/ RUSMATE604I	I	Colorings of graph	2.5	3
	II	Planar graph		
	III	Combinatorics		
Number Theory and its Applications II (Elective II)				
RUSMATE604II/ RUAMATE604II	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pells Equation, Arithmetic Functions, Special Numbers		
Course		**Practicals**	**Credits**	**L/Week**
RUSMATP601/ RUAMATP601		Practicals based on RUSMAT601/RUAMAT601 and RUSMAT602/RUAMAT602	3	6
RUSMATP602/ RUAMATP602		Practicals based on RUSMAT603/RUAMAT603, USMTE604I/UAMTE604I or USMTE604II/UAMTE604II	3	6

T.Y.B.Sc Mathematics
Semester V

Course: Integral Calculus

Course Code: RUSMAT501 / RUAMAT501

Learning Objectives:

1. Introduce notion of Multiple integrals.
2. To introduce notion of surface integrals, line integrals and their applications to Physics.

Learning Outcomes:

1. Learner will be able to apply concepts of multiple integrals in the field of physics.
2. Learner will be able to apply concepts of line integrals in the field of physics.
3. Learner will be able to apply concepts of surface integrals in the field of physics.

Detailed Syllabus

Unit I: Multiple Integrals (15 Lectures)

Definition of double (respectively: triple) integral of a function bounded on a rectangle (respectively: box), Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Basic properties of double and triple integrals proved using the Fubini's theorem such as; Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions, Formulae for the integrals of sums and scalar multiples of integrable functions, Integrability of continuous functions. More generally, integrability of bounded functions having finite number of points of discontinuity, Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only), Polar, cylindrical and spherical coordinates and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

Unit II: Line Integrals (15 Lectures)

Review of Scalar and Vector fields on $\mathbb{R}^n$. Vector Differential Operators, Gradient Paths (parametrized curves) in $\mathbb{R}$ (emphasis on $\mathbb{R}$ and $\mathbb{R}$), Smooth and piecewise smooth paths, Closed paths, Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path, Basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters, Examples.

Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative, Green's Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

Unit III: : Surface Integrals (15 Lectures)

Parameterized surfaces. Smoothly equivalent parameterizations, Area of such surfaces. Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface. Curl and divergence of a vector field, Elementary identities involving gradient, curl and divergence. Stoke's Theorem (proof assuming the general form of Green's Theorem), Examples. Gauss' Divergence Theorem (proof only in the case of cubical domains), Examples.

Reference Books:

(1) T Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.

(2) R. Courant and F. John,, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.

(3) W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.

(4) M. H. Protter and C. B. Morrey, Jr., CIntermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.

(5) G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.

(6) D. V. Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989

(7) R Courant and F. John., Introduction to Calculus and Analysis, Vol I. Reprint of 1st Ed. Springer-Verlag, New York, 1999.

(8) Sudhir R. Ghorpade and Balmohan Limaye, A course in Multivariable Calculus and Analysis, Springer International Edition.

Course : Algebra II
Course Code : RUSMAT502/ RUAMAT502

Learning Objectives:

1. To introduce notion of Group and Subgroups
2. To introduce notion of direct product of groups.

Learning Outcomes:

1. Learner will be able to examine properties of groups and subgroups.
2. Learner will be able to identify normal subgroups.
3. Learner will be able to illustrate examples of direct product of groups.

Detailed Syllabus

Unit 1 : Group Theory

i. Groups, definition and properties, examples such as $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, GL_n(\mathbb{R})$, $SL_n(\mathbb{R})$, O_n (= the group of $n \times n$ real orthogonal matrices), B_n (= the group of $n \times n$ nonsingular upper triangular matrices), S_n, $\mathbb{Z}_n$, $U(n)$ the group of prime, residue classes modulo n under multiplication, Quarternion group, Dihedral group as group of symmetries of regular $n-$gon, abelian group, finite and infinite groups.

ii. Subgroups, necessary and sufficient condition for a non-empty subset of a group to be a subgroup. Examples, cyclic subgroups, centre $Z(G)$.

iii. Order of an element. Subgroup generated by a subset of the group. Cyclic group. Examples of cyclic groups such as $\mathbb{Z}$ and the group μ_n of the n–th roots of unity.

iv. Cosets of a subgroup in a group. Lagrange's Theorem.

v. Homomorphisms, isomorphisms, automorphisms, kernel and image of a homomorphism.

Unit 2 : Normal Subgroups

i. Normal subgroup of a group, centre of a group, Alternating group A_n, cycles, Quotient group.

ii. First Isomorphism Theorem, Second Isomorphism Theorem, Third Isomorphism Theorem, Correspondence Theorem.

iii. Permutation groups, cycle decomposition, Cayley's Theorem for finite groups..

iv. External direct product of groups, order of an element in a direct product, criterion for external product of finite cyclic groups to be cyclic.

v. Classification of groups of order ≤ 7

Unit 3 : Direct Product of Groups

i. Internal direct product of subgroups, H and K which are normal in G, such that $H \cap K = \{1\}$. If a group is internal direct product of two normal subgroups H and K and $HK = G$, it is isomorphic to the external direct product $H \times K$.

ii. Structure Theorem of finite abelian groups (statement only) and applications.

iii. Conjugacy classes in a group, class equation. A group of order p^2 is abelian.

Reference Books :

(1) I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.

(2) Michael Artin, Algebra, Prentice Hall of India, New Delhi.

(3) P.B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

(4) D. Dummit, R. Foote, Abstract Algebra, John Wiley and Sons, Inc.

Additional Reference Books :

(1) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

(2) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

(3) J. B. Fraleigh, A First Course in Abstract Algebra, Third edition, Narosa, New Delhi.

(4) T. W. Hungerford, Algebra, Springer.

Course: Topology of Metric Spaces
Course Code: RUSMAT503 /RUAMAT503

Learning Objectives:

1. To introduce notion of metric spaces, open sets closed sets in metric spaces
2. To introduce notion of continuity in metric spaces.

Learning Outcomes:

1. Learner will be able to construct examples of metrics.
2. Learner will be able to compare properties of open, closed intervals, sequences and completeness on R with an arbitrary metric space.
3. Learner will be able to compare properties of continuity on R with an arbitrary metric space.

Detailed Syllabus

Unit I: Metric Spaces (15 Lectures)

Definition, examples of metric spaces $\mathbb{R}$, $\mathbb{R}^2$ Euclidean space $\mathbb{R}^n$ sup and sum metric, $\mathbb{C}$ (complex numbers), normed spaces. distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces. Product of two metric spaces. Open balls and open sets in a metric space, examples of open sets in various metric spaces, Hausdorff property, interior of a set. Structure of an open set in $\mathbb{R}$, equivalent metrics. Distance of a point from a set, distance between sets, diameter of a set in a metric space and bounded sets.

Unit II: Closed sets, Sequences, Completeness (15 Lectures)

Closed ball in a metric space, Closed sets- definition, examples. Limit point of a set, Isolated point, A closed set contains all its limit points, Closure of a set and boundary, Sequences in a metric space, Convergent sequence in a metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces, $\mathbb{R}$ with different metrics and other metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces, Dense subsets in a metric space and Separability. Definition of complete metric spaces, Examples of complete metric spaces. Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's Intersection Theorem.

Unit III: Continuity (15 Lectures)

Epsilon-delta definition of continuity at a point of a function from one metric space to another. Equivalent characterizations of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space. Continuity of the composite of continuous functions.

Reference Books:

(1) S. Kumaresan, Topology of Metric spaces, Narosa, Second Edn.

(2) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

Additional Reference Books:

(1) W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

(2) T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

(3) P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.

(4) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

(5) D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis. Narosa, New Delhi

(6) G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.

(7) Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press, 2009

Course: Graph Theory (Elective I)
Course: Code RUSMATE504I / RUAMATE504I

Learning Objectives:

1. To introduce notion of Graph and its various attributes
2. To apply notion of graph to various branches of knowledge.

Learning Outcomes:

1. Learner will be able to apply the concepts of graphs and trees to the fields of chemistry, physics and biological sciences.
2. Learner will be able to apply the concepts of hamiltonian and eulerian to the fields of chemistry, physics and biological sciences.

Detailed Syllabus

Unit I: Basics of Graphs (15 Lectures)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem.

Unit II: Trees (15 Lectures)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of complete graphs, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs.

Unit III: Eulerian and Hamiltonian graphs (15 Lectures)

Eulerian graph and its characterization, Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G - S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs-Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of a graph and simple results.

Reference Books:

(1) Bondy and Murty, Graph Theory with Applications

(2) Balkrishnan and Ranganathan, Graph theory and applications.

(3) West D.B., Introduction to Graph Theory , Second ed., Prentice Hall 2001.

(4) Sharad Sane, Combinatorial Techniques, Hindustan Book Agency.

Additional Reference Books:

(1) Behzad and Chartrand , Graph theory

(2) Choudam S. A., Introductory Graph theory.

Course: Number Theory and its Applications (Elective II)
Course Code: RUSMATE504II / RUAMATE504II

Learning Objectives:

1. To introduce congruences and factorization
2. To introduce Dipophanine equations
3. To introduce Cryptography

Learning Outcomes:

1. Learner will be able to understand various aspects of factorization
2. Lerner will be able to understand importance of cryptography in todays world.

Detailed Syllabus

Unit 1 : Congruences and Factorization
Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m, Euler's function and its properties, Fermat's Little Theorem, Euler's generation of Fermat's Little Theorem, Wilson's Theorem, Linear congruence, The Chinese Remainder Theorem, Congruence of higher degree, The Fermat-Kraitchik Factorization Method.

Unit 2 : Diophantine Equations and their Solutions
The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$ where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x, y, z) with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples –section 5.4 of Number theory by Niven-Zuckermann-Montgomery.

Unit 3 : Primitive Roots and Cryptography
Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference Books :

(1) David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.

(2) Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley and Sons. Inc.

(3) M. Artin, Algebra. Prentice Hall.

(4) K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

Course: Practicals (Based on RUSMAT501 / RUAMAT501 and RUSMAT502 / RUAMAT502)

Course Code: RUSMATP501 / RUAMATP501

Suggested Practicals (Based on RUSMAT501 / RUAMAT501)

(1) Evaluation of double and triple integrals.
(2) Change of variables in double and triple integrals and applications.
(3) Line integrals of scalar and vector fields
(4) Green's theorem, conservative field and applications
(5) Evaluation of surface integrals
(6) Stoke's and Gauss divergence theorem
(()7) Miscellaneous theory questions.

Suggested Practicals (Based on RUSMAT502 / RUAMAT502)

(1) Examples and properties of groups
(2) Group of symmetry of equilateral triangle, rectangle, square.
(3) Subgroups
(4) Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
(5) Left and right cosets of a subgroup, Lagrange's Theorem.
(6) Group homomorphisms, isomorphisms.
(7) Miscellaneous theory questions.

Practicals
(Based on RUSMAT503/RUAMAT503, RUSMATE504I/RUAMATE504I and RUSMATE504II/RUAMATE504II)

Course Code: RUSMATP502/RUAMATP502

Suggested Practicals (Based on RUSMAT503/RUAMAT503)

(1) Examples of Metric Spaces.

(2) Open balls and Open sets in Metric / Normed Linear spaces, Interior Points.

(3) Subspaces, Closed Sets and Closure, Equivalent Metrics and Norms.

(4) Sequences, Convergent and Cauchy Sequences in a Metric Space, Complete Metric Spaces, Cantors Intersection Theorem and its Applications.

(5) Continuous Functions on Metric Spaces

(6) Characterization of continuity at a point in terms of metric spaces.

(7) Miscellaneous Theory Questions.

Suggested Practicals (Based on RUSMATE504I/RUAMATE504I)

(1) Handshaking Lemma and Isomorphism.

(2) Degree sequence.

(3) Trees, Cayley Formula.

(4) Applications of Trees.

(5) Eulerian Graphs.

(6) Hamiltonian Graphs.

(7) Miscellaneous Problems.

Suggested Practicals (Based on RUSMATE504II / RUAMATE504II)

(1) Congruences.

(2) Linear congruences and congruences of higher degree.

(3) Linear diophantine equations.

(4) Pythagorean triples and sum of squares.

(5) Cryptosystems (Private Key).

(6) Cryptosystems (Public Key) and primitive roots.

(7) Miscellaneous theoretical questions.

SEMESTER VI
Course: Basic Complex Analysis
Course Code: RUSMAT601 / RUAMAT601

Learning Objectives:

1. To introduce Complex Numbers , their subsets and complex-valued functions.
2. To introduce Mobius Transformations and singularities of sets of complex numbers.

Learning Outcomes:

1. Learner will be able to elaborate on properties of complex numbers.
2. Learner will be able to elaborate on properties of Mobius transforms and singularities in subsets of C.

Detailed Syllabus

Unit I: Complex Numbers and Functions of Complex variables (15 Lectures)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivr's formula, $\mathbb{C}$ as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane.

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions $f : \mathbb{C} \to \mathbb{C}$ real and imaginary part of functions, continuity at a point and algebra of continuous functions.

Unit II: Holomorphic functions (15 Lectures)

Derivative of $f : \mathbb{C} \to \mathbb{C}$; comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, $'$, f, g analytic then $f + g$, $f - g$, fg, f/g are analytic. chain rule.Theorem: If $f' = 0$ everywhere in a domain G then f must be constant throughout, Harmonic functions and harmonic conjugate.

Explain how to evaluate the line integral $\int f(z)dz$ over $|z - z_0| = r$ and prove the Cauchy integral formula: If f is analytic in $B(z_0, r)$ then for any w in $B(z_0, r)$ we have $f(w) = \displaystyle\int \frac{f(z)}{w - z}dz$ over $|z - z_0| = r$.

Unit III: Complex power series (15 Lectures)

Taylor's theorem for analytic functions, Mobius transformations –definition and examples. Exponential function, its properties, trigonometric function, hyperbolic functions, Power series of complex numbers and related results , radius of convergences, disc of convergence, uniqueness of series representation, examples.

Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, statement of residue theorem and calculation of residue.

Reference Books:

(1) J. W. Brown and R.V. Churchill, Complex analysis and Applications.

(2) S. Ponnusamy, Foundations Of Complex Analysis, Second Ed., Narosa, New Delhi. 1947

(3) R. E. Greene and S. G. Krantz, Function theory of one complex variable

(4) T. W. Gamelin, Complex analysis

Course: Algebra III
Course Code: RUSMAT602 / RUAMAT602

Semester VI
Course : Algebra III
Course Code : RUSMAT602/ RUAMAT602

Learning Objectives:

1. To introduce notion of Ring and ideal
2. To introduce factorization in commutative rings.
3. To introduce constructible numbers.

Learning Outcomes:

1. Learner will be able to extend concept of normal subgroup to ideal of the ring R.
2. Learner will be able to elaborate properties of ED, PID and UFD.
3. Learner will be able to find quadratic extensions of field F.

Detailed Syllabus

Unit 1 : Ring Theory

i. Ring (definition should include the existence of a unity element), zero divisor, unit, the multiplicative group of units of a ring. Basic properties and examples of rings.

ii. Commutative ring, integral domain, division ring, subring, examples, Characteristic of a ring, characteristic of an Integral Domain.

iii. Ring homomorphism, kernel of ring homomorphism, ideals, operations on ideals and quotient rings, examples.

iv. Factor theorem and First and Second isomorphism theorems for rings, Correspondence theorem for rings.

Unit 2 : Factorization

i. Principal ideal, maximal ideal, prime ideal, characterization of prime and maximal ideals in terms of quotient rings.

ii. Polynomial rings, R$[X]$ when R is an integral domain/ field, Eisenstein's criterion for irreducibility of a polynomial over $\mathbb{Z}$, Gauss lemma, prime and maximal ideals in polynomial rings.

iii Notions of euclidean domain (ED), principal ideal domain (PID) and unique factorization domain (UFD). Relation between these three notions (ED $\Rightarrow$ PID $\Rightarrow$ UFD).

iv Example of ring of Gaussian integers.

Unit 3 : Field Theory

i. Review of field, characteristic of a field, Characteristic of a finite field is prime.

ii. Prime subfield of a field, Prime subfield of any field is either $\mathbb{Q}$ or $\mathbb{Z}_p$ (upto isomorphism).

iii. Field extension, Degree of field extension. Algebraic elements, Any homomorphism of a field is injective.

iv. Any irreducible polynomial $p(x)$ over a field F has a root in an extension of the field, moreover the degree of this extension $\frac{F(x)}{(p(x))}$ over the field F is the degree of the polynomial $p(x)$.

v. The extension $\frac{\mathbb{Q}[x]}{(x^2-2)}$ *i.e.* $\mathbb{Q}(\sqrt{2})$, $\frac{\mathbb{Q}[x]}{(x^3-2)}$ *i.e.* $\mathbb{Q}(\sqrt[3]{2})$, $\frac{\mathbb{Q}[x]}{(x^2+1)}$ *i.e.* $\mathbb{Q}(i)$, Quadratic extensions of a field F when characteristic of F is not 2.

Reference Books :

(1) I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.

(2) Michael Artin, Algebra, Prentice Hall of India, New Delhi.

(3) P.B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

(4) D. Dummit, R. Foote, Abstract Algebra, John Wiley and Sons, Inc.

Additional Reference Books :

(1) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

(2) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

(3) J. B. Fraleigh, A First Course in Abstract Algebra, Third edition, Narosa, New Delhi.

(4) T. W. Hungerford, Algebra, Springer.

Course: Metric Topology

Course Code: RUSMAT603 /RUAMAT603

Learning Objectives:

1. To introduce notion of compactness and connectedness in Metric Spaces.
2. To introduce sequences and series of functions

Learning Outcomes:

1. Learner will be able to compare properties of compact and connected sets on R with an arbitrary metric spaces.
2. Learner will be able to elaborate on properties of sequences and series of functions.

Detailed Syllabus

Unit I: Compact Sets (15 Lectures)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$ and other metric spaces. Properties of compact sets:compact set is closed and bounded, every infinite bounded subset of a compact metric space has a limit point, Heine Borel theorem-every subset of Euclidean metric space $\mathbb{R}$ is compact if and only if it is closed and bounded. Equivalent statements for compact sets in $\mathbb{R}$; Heine-Borel property, Closed and boundedness property, Bolzano-Weierstrass property, Sequentially compactness property. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete.

Unit II: Connected sets (15 Lectures)

Separated sets- definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space. Connected subsets of $\mathbb{R}$, A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected, Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from b to $< 1, -1 >$ is a constant function. Path connectedness in $\mathbb{R}$, definition and examples, A path connected subset of $\mathbb{R}$ is connected, convex sets are path connected, Connected components, An example of a connected subset of $\mathbb{R}$ which is not path connected.

Unit III: Sequence and series of functions (15 Lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples. Consequences of these properties for series of functions, term by term differentiation and integration. Power series in $\mathbb{R}$ centered at origin and at some point $4F$ in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Reference Books:

(1) S. Kumaresan, Topology of Metric spaces. Narosa, Second Edn.

(2) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

(3) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

Additional Reference Books:

(1) W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

(2) T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

(3) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

(4) P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.

(5) D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis. Narosa, New Delhi

(6) G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.

(7) Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press, 2009

Course: Graph Theory and Combinatorics (Elective I)
Course Code: RUSMATE604I /RUAMATE604I

Learning Objectives:

1. To introduce colorings of graphs and its applications in various fields of knowledge
2. To introduce a few combinatorial methods and its applications.

Learning Outcomes:

1. Learner will be able to apply the concepts of colorings of graphs and planar graph in the fields of chemistry, physics and biological sciences.
2. Learner will be able to apply the concepts of combinatorics in the field of statistics.

Detailed Syllabus

Unit I: Colorings of graphs (15 Lectures)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and Edge cuts vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II: Planar graphs (15 Lectures)

Definition of planar graph. Euler formula and its consequences. Non planarity of K_5; $K(3;3)$. Dual of a graph. Polyhedran in $\mathbb{R}$ and existence of exactly five regularpolyhedra- (Platonic solids) Colorabilty of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem.

Unit III: Combinatorics (15 Lectures)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems Introduction to partial fractions and using Newton's binomial theorem for real power series, expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR.

Reference Books:

(1) Bondy and Murty, Graph Theory with Applications

(2) Balkrishnan and Ranganathan, Graph theory and applications.

(3) West D G, Graph Theory

(4) Richard Brualdi, Introduction to Combinatorics.

(5) Sharad Sane, Combinatorial Techniques, Hindustan Book Agency.

Additional Reference Books:

(1) Behzad and Chartrand , Graph theory

(2) Choudam S. A., Introductory Graph theory.

(3) Cohen, Combinatorics

Course: Number Theory and its Applications II (Elective II)

Course Code: RUSMATE604II / RUAMATE604II

Learning Objectives:

1. To introduce quadratic reciprocity.
2. To introduce Continued Fractions.
3. To introduce spacial numbers in Number Theory

Learning Outcomes:

1. Learner will be able to apply Gauss Lemma in different situations.
2. Learner willbe able to undertsand continuied fractions.
3. Learner will be able to understand and apply theoy of arithmetic functions in simple situations.

Detailed Syllabus

Unit 1 : Quadratic Reciprocity
Quadratic Residues and Legendre Symbol, Euler's criterion, Gauss's Lemma, Quadratic Reciprocity Law. The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit 2 : Continued Fractions
Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit 3 : Pell's Equation, Arithmetic Functions and Special Numbers
Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $T(n)$), $\sigma(n)$, $\sigma k(n)$, $w(n)$ and their properties, $\mu(n)$ and the Mobius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Reference Books :

(1) David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.

(2) Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley and Sons. Inc.

(3) M. Artin, Algebra. Prentice Hall.

(4) K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

Course: Practicals
(Based on RUSMAT601/RUAMT601 and RUSMAT602/RUAMAT602)

Course Code: RUSMATP601/RUAMATP601

Suggested Practicals (Based on RUSMAT601/RUAMAT601)

(1) Complex Numbers, subsets of $\mathbb{C}$ and their properties.
(2) Limits and continuity of complex-valued functions .
(3) Derivatives of functions of complex variables, analytic functions.
(4) Finding harmonic conjugate, Mobius transformations and Contour Integration.
(5) Cauchy integral formula, Taylor series, power series.
(6) Finding isolated singularities- removable, pole and essential, Laurent series, Calculation of residue.
(7) Miscellaneous theory questions.

Suggested Practicals (Based on RUSMAT602 / RUAMAT602)

(1) Rings, Subrings
(2) Ideals, Ring Homomorphism and Isomorphism
(3) Polynomial Rings
(4) Prime and Maximal Ideals
(5) Fields, Subfields
(6) Field Extensions
(7) Miscellaneous Theory Questions

Practicals
(Based on RUSMAT603/RUAMAT603 and RUSMATE604I/RUAMATE604I and RUSMATE604II/RUAMATE604II)

Course Code: RUSMATP602/RUAMATP602

Suggested Practicals (Based on RUSMAT603 / RUAMAT603)

(1) Examples of compact metric spaces.

(2) Equivalent conditions for a subset of a metric space to be compact

(3) Connectedness

(4) Path Connectedness

(5) Pointwise and uniform convergence of sequence and series of functions and their properties.

(6) Power series and Elementary functions.

(7) Miscellaneous Theory Questions.

Suggested Practicals (Based on RUSMATE604I / RUAMATE604I)

(1) Coloring of Graphs.

(2) Chromatic polynomials and connectivity.

(3) Planar graphs.

(4) Generating Functions.

(5) Rook polynomial.

(6) System of Distinct Representatives.

(7) Miscellaneous Problems.

Suggested Practicals (Based on RUSMATE604II / RUAMATE604II)

(1) Legendre Symbol.

(2) Jacobi Symbol and Quadratic congruences with composite moduli.

(3) Finite continued fractions.

(4) Infinite continued fractions.

(5) Pell's equations and Arithmetic functions of number theory.

(6) Special Numbers.

(7) Miscellaneous theoretical questions.

MODALITY OF ASSESSMENT

Theory Examination Pattern:
A) Internal Assessment - 40% :

Total: 40 marks.
1 One Assignment/Case study/Project/ seminars/presentation: 10 marks
2 One class Test (multiple choice questions / objective) 20 marks
3 Active participation in routine class instructional deliveries and Overall conduct as a responsible student, manners, skill in articulation, leadership qualities demonstrated through organizing co-curricular activities, etc. 10 marks

B) External examination - 60 %
Semester End Theory Assessment - 60 marks

i. Duration - These examinations shall be of 2 hours duration.

ii. Paper Pattern:

1. There shall be 3 questions each of 20 marks. On each unit there will be one question.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Options	Marks	Questions on
Q.1)A)	Any 1 out of 2	08	Unit I
Q.1)B)	Any 2 out of 4	12	
Q.2)A)	Any 1 out of 2	08	Unit II
Q.2)B)	Any 2 out of 4	12	
Q.3)A)	Any 1 out of 2	08	Unit III
Q.3)B)	Any 2 out of 4	12	

Practical Examination Pattern:
(A)Internal Examination:

Journal 05 marks
Test 15 marks
Total 20

(B) External (Semester end practical examination):
Particulars:
Test: 30 marks

PRACTICAL BOOK/JOURNAL

- The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.
- In case of loss of Journal and/ or Report, a Lost Certificate should be obtained from Head/ Co-ordinator / Incharge of the department; failing which the student will not be allowed to appear for the practical examination.

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