Res No: AB/II(18-19).2.RUA13 / AC/II(18-19).2.RUS8

S.P.Mandali's
RAMNARAIN RUIA AUTONOMOUS COLLEGE, MUMBAI-19

RUIA COLLEGE

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SYLLABUS FOR F.Y.B.Sc /F.Y.B.A

PROGRAM: B.Sc / B.A

COURSE: MATHEMATICS (RUSMAT/RUAMAT)

(Credit Based Semester and Grading System with effect from the academic year 2019–2020)

Semester I

Calculus I				
Course Code	**Unit**	**Topics**	**Credits**	**L/Week**
RUSMAT101,RUAMAT101	Unit I	Real Number System	3	3
	Unit II	Sequences		
	Unit III	Limits & Continuity		
Algebra I				
RUSMAT102	Unit I	Integers & Divisibility	3	3
	Unit II	Functions & Equivalence relations		
	Unit III	Polynomials		

Semester II

Calculus II				
Course Code	**Unit**	**Topics**	**Credits**	**L/Week**
RUSMAT201	Unit I	Continuity of a function on an interval	3	3
	Unit II	Differentiability and its applications		
	Unit III	Series		
Linear Algebra I				
RUSMAT202, RUAMAT201	Unit I	System of Linear Equations & Matrices	3	3
	Unit II	Vector Spaces		
	Unit III	Bases & Linear transformations		

Teaching Pattern

1. Three lectures per week per course. Each lecture is of 1 hour duration.
2. One tutorial per week per course (the batches to be formed as prescribed by the University)

Syllabus for Semester I & II

(RUSMAT101/RUAMAT101) CALCULUS I

Learning Objectives:

1. To introduce the learner to the properties of real number line.
2. To introduce sequences of real numbers.
3. To introduce notion of limit of a real valued function of one variable and continuity of real valued functions at a given point.

Learning Outcomes:

1. Learner will be able to explain the properties of real numbers.
2. Learner will be able to explain the notions of convergent sequences.
3. Learner will be able to outline the concepts of limits and continuity.
4. Learner will be able to apply the concepts of limits and continuity in the fields of economics, physics and biological sciences.

Detailed Syllabus:

Unit I: Real Number System (15 Lectures)

Real number system R and order properties of R, Absolute value |.| and its properties.

Bounded sets, statement of l.u.b. axiom, g.l.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals, Cantors nested interval theorem.

AM-GM inequality, Cauchy-Schwarz inequality, intervals and neighbourhoods, Hausdorff property.

Unit II: Sequences (15 Lectures)

Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, Limit of a convergent sequence and uniqueness of limit, Divergent sequences. Algebra of convergent sequences, sandwich theorem.

Convergence of standard sequences like

$$\left(\frac{1}{1+na}\right) \ \forall\ a>0, \quad (b^n), \quad |b|<1, \quad \left(c^{1/n}\right)\ \forall\ c>0 \text{ and } \left(n^{1/n}\right),$$

monotone sequences,convergence of monotone bounded sequence theorem and consequences such as convergence of $\left(\left(1+\dfrac{1}{n}\right)^n\right)$.

Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit. Every sequence in R has a monotonic subsequence. Bolzano-Weierstrass Theorem. Definition of a Cauchy sequence, every convergent sequence is a Cauchy sequence.

Unit III: : Limits and Continuity (15 Lectures)

Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function.

Graphs of some standard functions such as $|x|$, e^x, $\log x$, ax^2+bx+c, $\frac{1}{x}$, x^n $(n \geq 3)$, $\sin x$, $\cos\_x$, $\tan\_x$, $x\sin\_\left(\frac{1}{x}\right)$, $x^2\sin\_\left(\frac{1}{x}\right)$ over suitable intervals of $\mathbb{R}$.

$\varepsilon - \delta$ definition of limit of a real valued function of real variable. Evaluation of limit of simple functions using the definition, uniqueness limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand limit $\lim\limits_{x\to a^-} \_ f(x)$, right-hand limit $\lim\limits_{x\to a^+} \_ f(x)$, non existence of limits, $\lim\limits_{x\to -\infty} \_ f(x)$, $\lim\limits_{x\to \infty} \_ f(x)$ and $\lim\limits_{x\to a} \_ f(x) = \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

(3) R.G. Bartle, D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

(4) T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd, 1991.

(5) R. Courant, F. John, A Introduction to Calculus and Analysis, Volume I, Springer.

(6) A. Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(7) J. Stewart, Calculus, Third Edition, Brooks/Cole Publishing Company, 1994.

(8) S. R. Ghorpade, B. V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2006.

Tutorials for RUSMAT101, RUAMAT101:

1) Application based examples of Archimedean property, intervals, neighbourhood.
2) Consequences of l.u.b. axiom, infimum and supremum of sets.
3) Calculating limits of sequences.
4) Cauchy sequences, monotone sequences.

5) Limit of a function and Sandwich theorem.

6) Continuous and discontinuous functions.

(RUSMAT102) ALGEBRA I

Learning Objectives:

1. To introduce notion of divisibility of integers.
2. To introduce notion of equivalence relations.
3. To introduce notion of polynomials.

Learning Outcomes:

1. Learner will be able to experiment with divisibility of integers.
2. Learner will be able to explain concept of functions and equivalence relations.
3. Learner will be able to explain the properties of polynomials over R and C.

Detailed Syllabus

Prerequisites:
Set theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Permutations ${}^nP_r$ and Combinations ${}^nC_r$.

Complex numbers: Addition and multiplication of complex numbers, modulus, argument and conjugate of a complex number. , De Moivere's theorem.

Unit I: Integers and divisibility (15 Lectures)

Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal's Triangle.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b, and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruence relation: definition and elementary properties. Euler's ϕ function, Statements of Euler's theorem, Fermat's little theorem and Wilson's theorem, Applications.

Unit II: Functions and Equivalence relations (15 Lectures)

Definition of a relation, definition of a function; domain, co-domain and range of a function; composite functions, examples, image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion; Binary operation as a function, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of a partition of a set, every partition gives an equivalence relation and conversely.

Congruence modulo n is an equivalence relation on $\mathbb{Z}$; Residue classes and partition of $\mathbb{Z}$; Addition modulo n; Multiplication modulo n; examples.

Unit III: Polynomials (15 Lectures)

Definition of a polynomial, polynomials over the field F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

Division algorithm in $F[X]$, and g.c.d. of two polynomials and its basic properties, Euclidean algorithm, applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.

Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, A non constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number, n^{th} roots of unity, sum of all the n^{th} roots of unity.

Reference Books:

(1) D. M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

(2) N. L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

(3) I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

(4) G. Birkhoff and S. Maclane, A Survey of Modern Algebra, Third Edition, MacMillan, New York, 1965.

(5) N. S. Gopalkrishnan, University Algebra, New Age International Ltd, Reprint 2013.

(6) I. N. Herstein, Topics in Algebra, John Wiley, 2006.

(7) P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.

(8) K. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

(9) L Childs , Concrete Introduction to Higher Algebra, Springer, 1995.

Tutorials for RUSMAT102:

1) Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2) Division Algorithm and Euclidean algorithm in $\mathbb{Z}$, primes and the Fundamental Theorem of Arithmetic.
3) Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions.
4) Congruences and Eulers function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5) Equivalence relation.
6) Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

(RUSMAT201) CALCULUS II

Learning Objectives:

1. To introduce notion of differentialbility of a real valued function of one real variable.
2. To introduce notion of infinite series.

Learning Outcomes:

1. Learner will be able to analyze the properties of continuous functions.
2. Learner will be able to identify differentiable functions.
3. Learner will be able to analyze properties of differentiable functions.
4. Learner will be able to test the convergence of series.

Detailed Syllabus

Unit I: Continuity of a function on an interval (15 Lectures)

Review of the definition of continuity (at a point and on the domain). Uniform continuity, sequential continuity, examples.

Properties of continuous functions such as the following:

1. Intermediate value property
2. A continuous function on a closed and bounded interval is bounded and attains its bounds.
3. If a continuous function on an interval is injective then it is strictly monotonic and inverse function is continuous and strictly monotonic.
4. A continuous function on a closed and bounded interval is uniformly continuous.

Unit II: Differentiability and Applications (15 Lectures)

Differentiation of a real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

Chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples).

Rolle's Theorem, Lagrange's and Cauchy's mean value theorems, applications and examples

Taylor's theorem with Lagrange's form of remainder (without proof), Taylor polynomial and applications

Monotone increasing and decreasing function, examples

Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, concave, convex functions, points of inflection. Applications to curve sketching.

L'Hospital's rule without proof, examples of indeterminate forms.

Unit III: Series (15 Lectures)

Series $\sum_{n=1}^{\infty} a_n$ of real numbers, simple examples of series, Sequence of partial sums of a series, convergence of a series, convergent series, divergent series, Necessary condition: $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow a_n \to 0$, but converse is not true, algebra of convergent series, Cauchy criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$, Comparison test, limit comparison test, alternating series, Leibnitz's theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), Root test (without proof), and examples.

Reference Books:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) J. Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994

(3) T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia).

(4) R. Courant,F. John, A Introduction to Calculus and Analysis, Volume I, Springer.

(5) A. Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(6) S. R. Ghorpade, B. V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2006.

(7) K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

(8) G. B. Thomas, Calculus, 12th Edition, 2009.

Tutorials for RUSMAT201:

1) Calculating limit of series, Convergence tests.
2) Properties of continuous functions.
3) Differentiability, Higher order derivatives, Leibnitz theorem.

4) Mean value theorems and its applications.
5) Extreme values, increasing and decreasing functions.
6) Applications of Taylor's theorem and Taylor's polynomials.

(RUSMT202/RUAMAT201) LINEAR ALGEBRA

Learning Objectives:

1. To introduce system of linear equations and matrices
2. To introduce notion of vecror spaces and linear transformations.

Learning Outcomes:

1. Learner will be able to experiment with the system of linear equations and matrices.
2. Learner will be able to identify vector spaces.
3. Learner will be able to explain properties of vector spaces and subspaces.
4. Learner will be able to construct basis for a given vector space.
5. Learner will be able to explain properties of linear transformation.

Detailed Syllabus

Unit I: System of Linear equations and Matrices (15 Lectures)

Parametric equation of lines and planes, system of homogeneous and non-homogeneous linear equations, solution of a system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$;

Matrices with real entries; addition, scalar multiplication and multiplication of matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrices, diagonal matrices, upper triangular matrices, lower triangular matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices; identities such as $(AB)^t = B^tA^t; (AB)^{-1} = B^{-1}A^{-1}$.

System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector Spaces (15 Lectures)

Definition of a real vector space, examples such as $\mathbb{R}^n$, $\mathbb{R}[X]$, $M_{m\times n}(\mathbb{R})$, space of all real valued functions on a nonempty set.

Subspace: definition, examples, lines, planes passing through origin as subspaces of $\mathbb{R}^2$, $\mathbb{R}^3$ respectively, upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$; $P_n(X) = \{a_0 + a_1X + \cdots + a_nX^n | a_i \in \mathbb{R}\ \forall\, i,\ 0 \le i \le n\}$ as a subspace of $\mathbb{R}[X]$, the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbb{R}^n$.

Properties of a subspace such as necessary and sufficient condition for a nonempty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other.

Linear combination of vectors in a vector space; the linear span $L(S)$ of a nonempty subset S of a vector space, S is a generating set for $L(S)$; $L(S)$ is a vector subspace of V; linearly independent/linearly dependent subsets of a vector space, examples

Unit III: Bases and Linear Transformations (15 Lectures)

Basis of a finite dimensional vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two bases of a vector space have the same number of elements, any set of n linearly independent vectors in an n dimensional vector space is a basis, any collection of $n+1$ linearly independent vectors in an n dimensional vector space is linearly dependent, if W_1, W_2 are two subspaces of a vector space V then $W_1 + W_2$ is a subspace of the vector space V of dimension $\dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$, extending any basis of a subspace W of a vector space V to a basis of the vector space V.

Linear transformations; kernel $\ker(T)$ of a linear transformation T, matrix associated with a linear transformation T, properties such as: for a linear transformation T, $\ker(T)$ is a subspace of the domain space of T and the image $\mathrm{Image}(T)$ is a subspace of the co-domain space of T. If V, W are real vector spaces with $\{v_1, v_2, \ldots, v_n\}$ a basis of V and $\{w_1, w_2, \ldots, w_n\}$ any vectors in W then there exists a unique linear transformation $T : V \to W$ such that $T(v_j) = w_j \quad \forall j\,, 1 \leq j \leq n$, Rank Nullity theorem (statement only) and examples.

Reference Books:

(1) S. Lang, Introduction to Linear Algebra, Second Edition, Springer, 1986.

(2) S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd, 2000.

(3) M. Artin, Algebra, Prentice Hall of India Private Limited, 1991.

(4) K. Hoffman and R. Kunze, Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.

(5) G. Strang, Linear Algebra and its applications, Thomson Brooks/Cole, 2006

(6) L. Smith, Linear Algebra, Springer Verlag, 1984.

(7) A. R. Rao and P. Bhima Sankaran, Linear Algebra, TRIM 2nd Ed. Hindustan Book Agency, 2000.

(8) T. Banchoff and J. Warmers, Linear Algebra through Geometry, Springer Verlag, New York, 1984.

(9) S. Axler, Linear Algebra done right, Springer Verlag, New York, 2015.

(10) K. Janich, Linear Algebra, Springer Verlag New York, Inc. 1994.

(11) O. Bretcher, Linear Algebra with Applications, Pearson 2013.

(12) G. Williams, Linear Algebra with Applications. Jones and Bartlett Publishers, Boston, 2001.

Tutorials for RUSMAT202/RUAMAT201:

(1) Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$; row echelon form.

(2) Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.

(3) Verifying whether given $(V, +, .)$ is a vector space with respect to addition $+$ and scalar multiplication $.$

(4) Linear span of a non empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.

(5) Finding basis of a vector space such as $P_3[X]$, $M_3(\mathbb{R})$ etc. verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.

(6) Verifying whether a map $T : X \to Y$ is a linear transformation, finding kernel of a linear transformation and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

MODALITY OF ASSESSMENT

Theory Examination Pattern:

A) Internal Assessment - 40% :

Total: 40 marks.

1 One Assignment/Case study/Project/ seminars/presentation: 10 marks

2 One class Test (multiple choice questions / objective) 20 marks

3 Active participation in routine class instructional deliveries and Overall conduct as a responsible student, manners, skill in articulation, leadership qualities demonstrated through organizing co-curricular activities, etc. 10 marks

B) External examination - 60 %

Semester End Theory Assessment - 60 marks

i. Duration - These examinations shall be of 2 hours duration.

ii. Paper Pattern:

1. There shall be 3 questions each of 20 marks. On each unit there will be one question.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Options	Marks	Questions on
Q.1)A)	Any 1 out of 2	08	Unit I
Q.1)B)	Any 2 out of 4	12	
Q.2)A)	Any 1 out of 2	08	Unit II
Q.2)B)	Any 2 out of 4	12	
Q.3)A)	Any 1 out of 2	08	Unit III
Q.3)B)	Any 2 out of 4	12	