Res No: AB/II(18-19).2.RUA13 / AC/II(18-19).2.RUS8

S.P.Mandali's
RAMNARAIN RUIA AUTONOMOUS COLLEGE, MUMBAI-19

SYLLABUS FOR F.Y.B.Sc /F.Y.B.A

PROGRAM: B.Sc / B.A

COURSE: MATHEMATICS (RUSMAT/RUAMAT)

(Credit Based Semester and Grading System with effect from the academic year 2019–2020)

Semester I

Calculus I				
Course Code	**Unit**	**Topics**	**Credits**	**L/Week**
RUSMAT101,RUAMAT101	Unit I	Real Number System	3	3
	Unit II	Sequences		
	Unit III	Limits & Continuity		
Algebra I				
RUSMAT102	Unit I	Integers & Divisibility	3	3
	Unit II	Functions & Equivalence relations		
	Unit III	Polynomials		

Semester II

Calculus II				
Course Code	**Unit**	**Topics**	**Credits**	**L/Week**
RUSMAT201	Unit I	Continuity of a function on an interval	3	3
	Unit II	Differentiability and its applications		
	Unit III	Series		
Linear Algebra I				
RUSMAT202, RUAMAT201	Unit I	System of Linear Equations & Matrices	3	3
	Unit II	Vector Spaces		
	Unit III	Bases & Linear transformations		

Teaching Pattern

1. Three lectures per week per course. Each lecture is of 1 hour duration.
2. One tutorial per week per course (the batches to be formed as prescribed by the University)

Syllabus for Semester I & II

(RUSMAT101/RUAMAT101) CALCULUS I

Learning Objectives:

1. To introduce the learner to the properties of real number line.
2. To introduce sequences of real numbers.
3. To introduce notion of limit of a real valued function of one variable and continuity of real valued functions at a given point.

Learning Outcomes:

1. Learner will be able to explain the properties of real numbers.
2. Learner will be able to explain the notions of convergent sequences.
3. Learner will be able to outline the concepts of limits and continuity.
4. Learner will be able to apply the concepts of limits and continuity in the fields of economics, physics and biological sciences.

Detailed Syllabus:

Unit I: Real Number System (15 Lectures)

Real number system R and order properties of R, Absolute value $|.|$ and its properties.

Bounded sets, statement of l.u.b. axiom, g.l.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals, Cantors nested interval theorem.

AM-GM inequality, Cauchy-Schwarz inequality, intervals and neighbourhoods, Hausdorff property.

Unit II: Sequences (15 Lectures)

Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, Limit of a convergent sequence and uniqueness of limit, Divergent sequences. Algebra of convergent sequences, sandwich theorem.

Convergence of standard sequences like

$$\left(\frac{1}{1+na}\right)\ \forall\ a>0,\quad (b^n),\quad |b|<1,\quad \left(c^{1/n}\right)\ \forall\ c>0 \text{ and } \left(n^{1/n}\right),$$

monotone sequences,convergence of monotone bounded sequence theorem and consequences such as convergence of $\left(\left(1+\frac{1}{n}\right)^n\right)$.

Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit. Every sequence in R has a monotonic subsequence. Bolzano-Weierstrass Theorem. Definition of a Cauchy sequence, every convergent sequence is a Cauchy sequence.

Unit III: : Limits and Continuity (15 Lectures)

Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function.

Graphs of some standard functions such as $|x|$, e^x, $\log x$, ax^2+bx+c, $\frac{1}{x}$, x^n $(n \geq 3)$, $\sin x$, $\cos x$, $\tan x$, $x\sin\left(\frac{1}{x}\right)$, $x^2\sin\left(\frac{1}{x}\right)$ over suitable intervals of $\mathbb{R}$.

$\varepsilon - \delta$ definition of limit of a real valued function of real variable. Evaluation of limit of simple functions using the definition, uniqueness limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand limit $\lim\limits_{x \to a^-} f(x)$, right-hand limit $\lim\limits_{x \to a^+} f(x)$, non existence of limits, $\lim\limits_{x \to -\infty} f(x)$, $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to a} f(x) = \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

(3) R.G. Bartle, D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

(4) T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd, 1991.

(5) R. Courant, F. John, A Introduction to Calculus and Analysis, Volume I, Springer.

(6) A. Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(7) J. Stewart, Calculus, Third Edition, Brooks/Cole Publishing Company, 1994.

(8) S. R. Ghorpade, B. V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2006.

Tutorials for RUSMAT101, RUAMAT101:

1) Application based examples of Archimedean property, intervals, neighbourhood.
2) Consequences of l.u.b. axiom, infimum and supremum of sets.
3) Calculating limits of sequences.
4) Cauchy sequences, monotone sequences.

5) Limit of a function and Sandwich theorem.

6) Continuous and discontinuous functions.

(RUSMAT102) ALGEBRA I

Learning Objectives:

1. To introduce notion of divisibility of integers.
2. To introduce notion of equivalence relations.
3. To introduce notion of polynomials.

Learning Outcomes:

1. Learner will be able to experiment with divisibility of integers.
2. Learner will be able to explain concept of functions and equivalence relations.
3. Learner will be able to explain the properties of polynomials over R and C.

Detailed Syllabus

Prerequisites:

Set theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Permutations ${}^nP_r$ and Combinations ${}^nC_r$.

Complex numbers: Addition and multiplication of complex numbers, modulus, argument and conjugate of a complex number. , De Moivere's theorem.

Unit I: Integers and divisibility (15 Lectures)

Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal's Triangle.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b, and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruence relation: definition and elementary properties. Euler's ϕ function, Statements of Euler's theorem, Fermat's little theorem and Wilson's theorem, Applications.

Unit II: Functions and Equivalence relations (15 Lectures)

Definition of a relation, definition of a function; domain, co-domain and range of a function; composite functions, examples, image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion; Binary operation as a function, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of a partition of a set, every partition gives an equivalence relation and conversely.

Congruence modulo n is an equivalence relation on $\mathbb{Z}$; Residue classes and partition of $\mathbb{Z}$; Addition modulo n; Multiplication modulo n; examples.

Unit III: Polynomials (15 Lectures)

Definition of a polynomial, polynomials over the field F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

Division algorithm in $F[X]$, and g.c.d. of two polynomials and its basic properties, Euclidean algorithm, applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.

Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, A non constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number, n^{th} roots of unity, sum of all the n^{th} roots of unity.

Reference Books:

(1) D. M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

(2) N. L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

(3) I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

(4) G. Birkhoff and S. Maclane, A Survey of Modern Algebra, Third Edition, MacMillan, New York, 1965.

(5) N. S. Gopalkrishnan, University Algebra, New Age International Ltd, Reprint 2013.

(6) I. N. Herstein, Topics in Algebra, John Wiley, 2006.

(7) P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.

(8) K. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

(9) L Childs , Concrete Introduction to Higher Algebra, Springer, 1995.

Tutorials for RUSMAT102:

1) Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2) Division Algorithm and Euclidean algorithm in $\mathbb{Z}$, primes and the Fundamental Theorem of Arithmetic.
3) Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions.
4) Congruences and Eulers function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5) Equivalence relation.
6) Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

(RUSMAT201) CALCULUS II

Learning Objectives:

1. To introduce notion of differentialbility of a real valued function of one real variable.
2. To introduce notion of infinite series.

Learning Outcomes:

1. Learner will be able to analyze the properties of continuous functions.
2. Learner will be able to identify differentiable functions.
3. Learner will be able to analyze properties of differentiable functions.
4. Learner will be able to test the convergence of series.

Detailed Syllabus

Unit I: Continuity of a function on an interval (15 Lectures)

Review of the definition of continuity (at a point and on the domain). Uniform continuity, sequential continuity, examples.

Properties of continuous functions such as the following:

1. Intermediate value property
2. A continuous function on a closed and bounded interval is bounded and attains its bounds.
3. If a continuous function on an interval is injective then it is strictly monotonic and inverse function is continuous and strictly monotonic.
4. A continuous function on a closed and bounded interval is uniformly continuous.

Unit II: Differentiability and Applications (15 Lectures)

Differentiation of a real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

Chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples).

Rolle's Theorem, Lagrange's and Cauchy's mean value theorems, applications and examples

Taylor's theorem with Lagrange's form of remainder (without proof), Taylor polynomial and applications

Monotone increasing and decreasing function, examples

Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, concave, convex functions, points of inflection. Applications to curve sketching.

L'Hospital's rule without proof, examples of indeterminate forms.

Unit III: Series (15 Lectures)

Series $\sum_{n=1}^{\infty} a_n$ of real numbers, simple examples of series, Sequence of partial sums of a series, convergence of a series, convergent series, divergent series, Necessary condition: $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow a_n \to 0$, but converse is not true, algebra of convergent series, Cauchy criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$, Comparison test, limit comparison test, alternating series, Leibnitz's theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), Root test (without proof), and examples.

Reference Books:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) J. Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994

(3) T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia).

(4) R. Courant,F. John, A Introduction to Calculus and Analysis, Volume I, Springer.

(5) A. Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(6) S. R. Ghorpade, B. V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2006.

(7) K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

(8) G. B. Thomas, Calculus, 12th Edition, 2009.

Tutorials for RUSMAT201:

1) Calculating limit of series, Convergence tests.
2) Properties of continuous functions.
3) Differentiability, Higher order derivatives, Leibnitz theorem.

4) Mean value theorems and its applications.

5) Extreme values, increasing and decreasing functions.

6) Applications of Taylor's theorem and Taylor's polynomials.

(RUSMT202/RUAMAT201) LINEAR ALGEBRA

Learning Objectives:

1. To introduce system of linear equations and matrices
2. To introduce notion of vecror spaces and linear transformations.

Learning Outcomes:

1. Learner will be able to experiment with the system of linear equations and matrices.
2. Learner will be able to identify vector spaces.
3. Learner will be able to explain properties of vector spaces and subspaces.
4. Learner will be able to construct basis for a given vector space.
5. Learner will be able to explain properties of linear transformation.

Detailed Syllabus

Unit I: System of Linear equations and Matrices (15 Lectures)

Parametric equation of lines and planes, system of homogeneous and non-homogeneous linear equations, solution of a system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$;

Matrices with real entries; addition, scalar multiplication and multiplication of matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrices, diagonal matrices, upper triangular matrices, lower triangular matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices; identities such as $(AB)^t = B^tA^t; (AB)^{-1} = B^{-1}A^{-1}$.

System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector Spaces (15 Lectures)

Definition of a real vector space, examples such as $\mathbb{R}^n$, $\mathbb{R}[X]$, $M_{m\times n}(\mathbb{R})$, space of all real valued functions on a nonempty set.

Subspace: definition, examples, lines, planes passing through origin as subspaces of $\mathbb{R}^2$, $\mathbb{R}^3$ respectively, upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$; $P_n(X) = \{a_0 + a_1X + \cdots + a_nX^n | a_i \in \mathbb{R}\ \forall i,\ 0 \leq i \leq n\}$ as a subspace of $\mathbb{R}[X]$, the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbb{R}^n$.

Properties of a subspace such as necessary and sufficient condition for a nonempty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other.

Linear combination of vectors in a vector space; the linear span $L(S)$ of a nonempty subset S of a vector space, S is a generating set for $L(S)$; $L(S)$ is a vector subspace of V; linearly independent/linearly dependent subsets of a vector space, examples

Unit III: Bases and Linear Transformations (15 Lectures)

Basis of a finite dimensional vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two bases of a vector space have the same number of elements, any set of n linearly independent vectors in an n dimensional vector space is a basis, any collection of $n+1$ linearly independent vectors in an n dimensional vector space is linearly dependent, if W_1, W_2 are two subspaces of a vector space V then $W_1 + W_2$ is a subspace of the vector space V of dimension $\dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$, extending any basis of a subspace W of a vector space V to a basis of the vector space V.

Linear transformations; kernel $\ker(T)$ of a linear transformation T, matrix associated with a linear transformation T, properties such as: for a linear transformation T, $\ker(T)$ is a subspace of the domain space of T and the image $\mathrm{Image}(T)$ is a subspace of the co-domain space of T. If V, W are real vector spaces with $\{v_1, v_2, \ldots, v_n\}$ a basis of V and $\{w_1, w_2, \ldots, w_n\}$ any vectors in W then there exists a unique linear transformation $T : V \to W$ such that $T(v_j) = w_j \;\; \forall j, 1 \leq j \leq n$, Rank Nullity theorem (statement only) and examples.

Reference Books:

(1) S. Lang, Introduction to Linear Algebra, Second Edition, Springer, 1986.

(2) S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd, 2000.

(3) M. Artin, Algebra, Prentice Hall of India Private Limited, 1991.

(4) K. Hoffman and R. Kunze, Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.

(5) G. Strang, Linear Algebra and its applications, Thomson Brooks/Cole, 2006

(6) L. Smith, Linear Algebra, Springer Verlag, 1984.

(7) A. R. Rao and P. Bhima Sankaran, Linear Algebra, TRIM 2nd Ed. Hindustan Book Agency, 2000.

(8) T. Banchoff and J. Warmers, Linear Algebra through Geometry, Springer Verlag, New York, 1984.

(9) S. Axler, Linear Algebra done right, Springer Verlag, New York, 2015.

(10) K. Janich, Linear Algebra, Springer Verlag New York, Inc. 1994.

(11) O. Bretcher, Linear Algebra with Applications, Pearson 2013.

(12) G. Williams, Linear Algebra with Applications. Jones and Bartlett Publishers, Boston, 2001.

Tutorials for RUSMAT202/RUAMAT201:

(1) Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$; row echelon form.

(2) Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.

(3) Verifying whether given $(V, +, .)$ is a vector space with respect to addition $+$ and scalar multiplication .

(4) Linear span of a non empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.

(5) Finding basis of a vector space such as $P_3[X]$, $M_3(\mathbb{R})$ etc. verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.

(6) Verifying whether a map $T : X \to Y$ is a linear transformation, finding kernel of a linear transformation and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

MODALITY OF ASSESSMENT

Theory Examination Pattern:

A) Internal Assessment - 40% :

Total: 40 marks.
1 One Assignment/Case study/Project/ seminars/presentation: 10 marks
2 One class Test (multiple choice questions / objective) 20 marks
3 Active participation in routine class instructional deliveries and Overall conduct as a responsible student, manners, skill in articulation, leadership qualities demonstrated through organizing co-curricular activities, etc. 10 marks

B) External examination - 60 %

Semester End Theory Assessment - 60 marks

i. Duration - These examinations shall be of 2 hours duration.

ii. Paper Pattern:

1. There shall be 3 questions each of 20 marks. On each unit there will be one question.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Options	Marks	Questions on
Q.1)A)	Any 1 out of 2	08	Unit I
Q.1)B)	Any 2 out of 4	12	
Q.2)A)	Any 1 out of 2	08	Unit II
Q.2)B)	Any 2 out of 4	12	
Q.3)A)	Any 1 out of 2	08	Unit III
Q.3)B)	Any 2 out of 4	12	

Res No: AB/II(18-19).2.RUA13 / AC/II(18-19).2.RUS8

S.P.Mandali's
RAMNARAIN RUIA AUTONOMOUS COLLEGE, MUMBAI-19

RUIA COLLEGE
Explore • Experience • Excel

SYLLABUS FOR S.Y.B.Sc /S.Y.B.A

PROGRAM: B.Sc / B.A

COURSE: MATHEMATICS (RUSMAT/RUAMAT)

(Credit Based Semester and Grading System with effect from the academic year 2019–2020)

Semester III

Course Code	Unit	Topics	Credits	L/Week
Calculus III				
RUSMAT301	Unit I	Riemann Integration	3	3
	Unit II	Applications of Integration		
	Unit III	Improper Integrals		
Linear Algebra II				
RUSMAT 302, RUAMAT 301	Unit I	Linear Transformations and Matrices	3	3
	Unit II	Determinants		
	Unit III	Inner Product Spaces		
Discrete Mathematics				
RUSMAT303, RUAMAT302	Unit I	Preliminary Counting	3	3
	Unit II	Permutations and Recurrence Relations.		
	Unit III	Advanced Counting		

Semester IV

Course Code	Unit	Topics	Credits	L/Week
Calculus of Several Variables				
RUSMAT401	Unit I	Functions of Several Variables	3	3
	Unit II	Differentiation		
	Unit III	Applications		
Linear Algebra III				
RUSMAT402, RUAMAT401	Unit I	Quotient Spaces and Orthogonal Linear Transformations	3	3
	Unit II	Eigenvalues and Eigenvectors		
	Unit III	Diagonalization		
Ordinary Differential Equations				
RUSMAT403, RUAMAT402	Unit I	First order ordinary differential equations	3	3
	Unit II	Second order ordinary differential equations		
	Unit III	Power Series Solutions of Ordinary differential Equations		

S.Y.B.Sc Mathematics
Semester III

RUSMAT301 CALCULUS III

Learning Objectives:

1. To introduce notion of Riemann integration and improper integrals.

Learning Outcomes:

1. Learner will be able to identify Riemann Integrable functions.
2. Learner will be able to analyze applications of integration.
3. Learner will be able to test the convergence of improper integrals.

Detailed Syllabus:

Note: Review of lim inf and lim sup.

Unit I: Riemann Integration(15 Lectures)

1. Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals.
2. Concept of Riemann integration, criterion for Riemann integrability
3. Properties of Riemann integrable functions.
4. Basic results on Riemann integration.
5. Indefinite integrals and its basic properties.

Unit II: Applications of Integration (15 Lectures)

1. Average value of a function, Mean Value Theorem of Integral Calculus
2. Area between the two curves.
3. Arc length of a curve.
4. Surface area of surfaces of revolution
5. Volumes of solids of revolution, washer method and shell method.
6. Definition of the natural logarithm $\ln x$ as $\int_1^x \frac{1}{t} dt$, $x > 0$, basic properties.
7. Definition of the exponential function $\exp x$ as the inverse of $\ln x$, basic properties.
8. Power functions with fixed exponent or with fixed base, basic properties.

Unit III: : Improper Integrals (15 Lectures)

1. Definitions of two types of improper integrals, necessary and sufficient conditions for convergence.

2. Absolute convergence, comparison and limit comparison test for convergence. Abel's and Dirichlet's tests.

3. Gamma and Beta functions and their properties.

Reference Books:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) A. Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(3) S. R. Ghorpade, B. V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd., 2000.

(4) T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd.

(5) T. M. Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi, 1974.

(6) J. Stewart, Calculus, Third Ed., Brooks/Cole Publishing Company, 1994.

(7) R. Courant, F. John, Introduction to Calculus and Analysis, Vol I. Reprint of 1st Ed. Springer-Verlag, New York, 1999.

(8) M. H. Protter, Basic Elements of Real Analysis, Springer-Verlag, New York, 1998.

(9) G.B. Thomas, R. L. Finney, Calculus and Analytic Geometry, Ninth Ed.(ISE Reprint), Addison-Wesley, Reading Mass, 1998.

(10) R.G. Bartle, D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

Suggested Tutorials:

(1) Calculation of upper sum, lower sum and Riemann integral.

(2) Problems on properties of Riemann integral.

(3) Sketching of regions in $\mathbb{R}^2$ and $\mathbb{R}^3$, graph of a function, level sets, conversions from one coordinate system to another.

(4) Applications to compute average value, area, volumes of solids of revolution, surface area of surfaces of revolution, moment, center of mass.

(5) Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.

(6) Problems on Gamma, Beta functions and properties

(RUSMAT302/RUAMAT301) LINEAR ALGEBRA II

Learning Objectives:

1. To introduce notion of dimensions of vector spaces and determinants.
2. Geometrical interpretation of the concept of determinants.
3. To introduce notion of inner product spaces.

Learning Outcomes:

1. Learner will be able to examine dimensions of vector spaces.
2. Learner will be able to explain the concept of determinants.
3. Learner will be able to apply the concept of determinants to geometry.
4. Learner will be able to identify inner product spaces.
5. Learner will be able to outline properties of inner products.

Detailed Syllabus:

Unit I: Linear Transformations and Matrices (15 Lectures)

1. Review of linear transformations, kernel and image of a linear transformation, Rank-Nullity theorem (with proof), linear isomorphisms, inverse of a linear isomorphism, any n-dimensional real vector space is isomorphic to $\mathbb{R}^n$.

2. The matrix units, row operations, elementary matrices and their properties.

3. Row Space, column space of $m \times n$ matrix, row rank and column rank of a matrix, equivalence of the row and column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an $m \times n$ matrix A and rank of the corresponding linear transformation, The dimension of solution space of the system of the linear equations $Ax = 0$

5. The solution of non-homogeneous system of linear equations represented by $Ax = b$, existence of a solution when $\text{rank}(A) = \text{rank}(A|b)$. The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an n-linear skew-symmetric function from $\mathbb{R}^n \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n \to \mathbb{R}$ such that determinant of $(E^1, E^2, \ldots, E^n)$ is 1, where E^j denote the j^{th} column of the $n \times n$ identity matrix I_n.

2. Existence and uniqueness of determinant function via permutations, Computation of determinant of 2×2, 3×3 matrices, diagonal matrices, basic results on determinants such as $\det\_(A^t) = \det\_(A)$, $\det\_(AB) = \det\_(A)\det\_(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular matrices and lower triangular matrices.

3. Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, the existence and uniqueness of the system $Ax = b$, where A is $n \times n$ matrix A, with $\det\_(A) \neq 0$, cofactors and minors, adjoint of an $n \times n$ matrix A, basic results such as $A.\text{Adj}(A) = \det(A)I_n$. An $n \times n$ real matrix A is invertible if and only if $\det\_(A) \neq 0$, $A^{-1} = \frac{1}{\det\_(A)}\text{Adj}(A)$ for an invertible matrix A, Cramer's rule.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in $\mathbb{R}^n$, Definition of an inner product on a vector space over $\mathbb{R}$, examples of inner product

2. Norm of a vector Cauchy-Schwarz inequality, triangle inequality, orthogonality of vectors, Pythagorus theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, the projection being the closest approximation, Orthogonal complements of a subspace, orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$, orthogonal sets and orthonormal sets in an inner product space, orthogonal and orthonormal bases, Gram-Schmidt orthogonalization process, simple examples in $\mathbb{R}^3$, $\mathbb{R}^4$.

Reference Books:

(1) S. Lang, Introduction to Linear Algebra, Springer Verlag, 1997

(2) S. Kumarasen, Linear Algebra A geometric approach, Prentice Hall of India Private Ltd, 2000

(3) M. Artin, Algebra, Prentice Hall of India Private Ltd. 1991

(4) K. Hoffman, R.Kunze, Linear algebra, Tata McGraw-Hill, New Delhi. 1971

(5) G. Strang, Linear Algebra and its applications, International student Edition. 2016

(6) L. Smith, Linear Algebra and Springer Verlag. 1978

(7) A. R. Rao and P.BhimaSankaran, Linear Algebra, Tata McGraw-Hill, New Delhi. 2000

(8) T. Banchoff, J. Wermer, Linear Algebra through Geometry, Springer Verlag New York, 1984.

(9) S. Axler , Linear Algebra done right, Springer Verlag, New York, 2015

(10) K. Janich , Linear Algebra, Springer, 1994

(11) O. Bretcher, Linear Algebra with Applications, Prentice Hall, 1996

(12) G. Williams, Linear Algebra with Applications, Narosa Publication, 1984

(13) H. Anton, Elementary Linear Algebra, Wiley, 2014.

Suggested Tutorials:

(1) Rank-Nullity Theorem

(2) System of linear equations

(3) Determinants, calculating determinants of 2×2; 3×3 matrices, $n \times n$ diagonal, upper triangular matrices using Laplace expansion

(4) Finding inverses of 3×3 matrices using adjoint. Verifying $A.\text{Adj}A = (\text{Det}A)I_3$

(5) Examples of inner product spaces and orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$.

(6) Gram-Schmidt method.

(RUSMAT303/RUAMAT302) DISCRETE MATHEMATICS

Learning Objectives:

1. To introduce notion of infinite sets and countability
2. To introduce notion of two way counting
3. To introduce notion of recurrence relations
4. to introduce notion of multisets.

Learning Outcomes:

1. Learner will be able to examine if given sets are countable.
2. Learner will be able to experiment with addition and multiplication principle.
3. Learner will be able to solve recurrence relations.
4. Learner will be able to extend notions of counting to multisets.

Detailed Syllabus:

Unit I: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets, examples such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$, $(0, 1)$, $\mathbb{R}$
2. Addition and multiplication principle, counting sets of pairs,two way counting, Permutation and Combination of sets.
3. Pigeonhole principle and its applications.

Unit II: Permutations and Recurrence relation (15 Lectures)

1. Permutation of objects, S_n composition of permutations, results such as every permutation is product of disjoint cycles, every cycle is product of transpositions, even and odd permutations, rank and signature of permutation, cardinality S_n, A_n.
2. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non homogeneous) recurrence relation by using iterative method, solving a homogeneous relation of second degree using algebraic method proving the necessary result.

Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following
   - $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
   - $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
   - $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
   - $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
2. Permutations and combinations of multi-sets, circular permutations, emphasis on solving problems.

3. Non-negative and positive integral solutions of the equation $x_1+x_2+\cdots+x_r = n$.

4. Principle of Inclusion and Exclusion, its applications, derangements, explicit formulae for d_n, various identities involving d_n, deriving formula for Euler's phi function $\phi(n)$

Reference Books:

(1) N. Biggs, Discrete Mathematics, Oxford University Press, 1985

(2) R. Brualdi, Introductory Combinatorics, Pearson, 2010.

(3) V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press, 1985

(4) A. Tucker, Applied Combinatorics, John Wiley and Sons,1980

(5) S. S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

Suggested Practicals (Tutorials for B.A.):

(1) Problems based on counting principles, two way counting.

(2) Pigeonhole principle.

(3) Signature of a permutation. Expressing permutation as the product of disjoint cycles. Inverse of a permutation.

(4) Recurrence relation.

(5) Multinomial theorem, identities, permutations and combinations of multisets.

(6) Inclusion-Exclusion principle, Derangements, Euler's phi function.

SEMESTER IV

(RUSMAT401) CALCULUS OF SEVERAL VARIABLES

Learning Objectives:

1. To introduce notion of real valued functions of several variables.
2. Surfaces and curves as real valued functions of several real variables.
3. To introduce diffrentability of real valued functions of several variables.

Learning Outcomes:

1. Learner will be able to compare properties of functions of several variables with those of functions of one variable.
2. Learner will be able to deduce geometrical properties of surfaces and lines.
3. Learner will be able to apply the concept of differentiability to other sciences.

Detailed Syllabus:

Unit I: Functions of several variables (15 Lectures)

1. Euclidean space, $\mathbb{R}^n$- norm, inner product, distance between two points, open ball in $\mathbb{R}^n$, definition of an open set / neighbourhood, sequences in $\mathbb{R}^n$, convergence of sequences (these concepts should be specifically discussed for $n = 2$ and $n = 3$).

2. Functions from $\mathbb{R}^n \to \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \to \mathbb{R}^n$ (Vector fields), sketching of regions in $\mathbb{R}^2$ and $\mathbb{R}^3$. Graph of a function, level sets, cartesian coordinates, polar coordinates, spherical coordinates, cylindrical coordinates and conversions from one coordinate system to other. Iterated limits, limits and continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity of components of vector fields.

3. Directional derivatives and partial derivatives of scalar fields.

4. Mean value theorem for derivatives of scalar fields.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point (in terms of linear transformation) and in an open set, total derivative, uniqueness of total derivative of a differentiable function at a point, basic results on continuity, differentiability, partial derivative and directional derivative.

2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.

3. Chain rule for scalar fields.

4. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Unit III: Applications (15 Lectures)

1. Second order Taylor's formula for scalar fields.

2. Differentiability of vector fields, definition of differentiability of a vector field at a point Jacobian and Hessian matrix, differentiability of a vector field at a point implies continuity, the chain rule for derivative of vector fields (statement only).

3. Mean value inequality.

4. Maxima, minima and saddle points.

5. Second derivative test for extrema of functions of two variables.

6. Method of Lagrange multipliers.

Reference Books:

(1) S. R. Ghorpade, B. V. Limaye, A Course in Multivariable Calculus and Analysis, Springer, 2010.

(2) T. Apostol, Calculus, Vol. 2, John Wiley, 1969.

(3) J. Stewart, Calculus, Brooke/Cole Publishing Co., 1994.

Suggested Tutorials:

(1) Sequences in $\mathbb{R}^2$ and $\mathbb{R}^3$, limits and continuity of scalar fields and vector fields, using definition and otherwise, iterated limits.

(2) Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.

(3) Total derivative, gradient, level sets and tangent planes.

(4) Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.

(5) Taylor's formula, differentiation of a vector field at a point, finding Jacobian and Hessian matrix, Mean value inequality.

(6) Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.

(RUSMAT402 / RUAMAT401) Linear Algebra III

Learning Objectives:

1. To introduce notion of quotient structures.
2. To introduce geometrical aspects of eigenvalues and eigenvectors of linear transformations and matrices.

Learning Outcomes:

1. Learner will be able to explain quotient structures on vector spaces.
2. Learner will be able to explain the concepts of orthogonalization.
3. Learner will be able to apply the concepts of eigenvalues and eigenvectors to geometry.

Detailed Syllabus:

Unit I: Quotient Spaces and Orthogonal Linear Transformations (15 Lectures)

(1) Review of vector spaces over $\mathbb{R}$, subspaces and linear transformations.

(2) Quotient spaces, first isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), dimension and basis of the quotient space V/W, where V is finite dimensional vector space and W is subspace of V.

(3) Orthogonal transformations, isometries of a real finite dimensional inner product space, translations and reflections with respect to a hyperplane, orthogonal matrices over $\mathbb{R}$, equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, orthogonal transformation of $\mathbb{R}$, any orthogonal transformation in $\mathbb{R}$ is a reflection or a rotation, characterization of isometries as composites of orthogonal transformations and translation.

(4) Characteristic polynomial of an $n \times n$ real matrix, Cayley Hamilton theorem and its applications (Proof assuming the result: $A\,\mathrm{Adj}(A) = \det(A)I_n$ for an $n \times n$ matrix A over the polynomial ring $\mathbb{R}[t]$).

Unit II: Eigenvalues and eigen vectors (15 Lectures)

(1) Eigen values and eigen vectors of a linear transformation $T : V \to V$ where V is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of $n \times n$ real matrices, linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation and a matrix.

(2) The characteristic polynomial of a $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, similar matrices, relation with change of basis, invariance of the characteristic polynomial and eigen values of similar matrices, every $n \times n$ square matrix with real eigenvalues is similar to an upper triangular matrix.

(3) Minimal Polynomial of a matrix, examples, diagonal matrix, similar matrix, invariant subspaces.

Unit III: Diagonalisation (15 Lectures)

(1) Geometric multiplicity and algebraic multiplicity of eigen values of an $n \times n$ real matrix, equivalent statements about diagonalizable matrix and multiplicities of its eigenvalues , examples of non diagonalizable matrices,

(2) Diagonalisation of a linear transformation $T : V \to V$ where V is a finite dimensional real vector space and examples.

(3) Orthogonal diagonalisation and quadratic forms, diagonalisation of real symmetric matrices, examples, applications to real quadratic forms, rank and signature of a real quadratic form

(4) Classification of conics in $\mathbb{R}^2$ and quadric surfaces in $\mathbb{R}^3$, positive definite and semi definite matrices, characterization of positive definite matrices in terms of principal minors.

Reference Books:

(1) S. Kumaresan, Linear Algebra: A Geometric Approach, Prentice Hall of India, 2000

(2) R. Rao, P. Bhimasankaram, Linear Algebra, TRIM, Hindustan Book Agency, 2000.

(3) T. Banchoff, J. Wermer, Linear Algebra through Geometry, Springer, 1992.

(4) L. Smith, Linear Algebra, Springer, 1978.

(6) K Hoffman, Kunze, Linear Algebra, Prentice Hall of India, New Delhi, 1971.

Suggested Tutorials:

(1) Quotient spaces, orthogonal transformations.

(2) Cayley Hamilton theorem and applications.

(3) Eigenvalues and eigenvectors of a linear transformation and a square matrix.

(4) Similar matrices, minimal polynomial.

(5) Diagonalization of a matrix.

(6) Orthogonal diagonalization and quadratic forms.

(RUSMAT403/ RUAMAT402) ORDINARY DIFFERENTIAL EQUATIONS

Learning Objectives:

1. To introduce notion of ordinary differential equations
2. To introduce simple methods to solve ODE
3. To introduce simple applications of ODE

Learning Outcomes:

1. Learner will be able to classify the ODE according to degree and order of ODE.
2. Learner will be able to solve an ODE.
3. Learner will be able to apply the concepts of ODE to biological sciences and physics.

Detailed Syllabus:

Unit I: First order First degree Differential equations (15 Lectures)

(1) Definition of a differential equation, order, degree, ordinary differential equation, linear and non linear ODE.

(2) Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only), Lipschitz function, examples

(3) Review of solution of homogeneous and non- homogeneous differential equations of first order and first degree, notion of partial derivative, exact equations, general solution of exact equations of first order and first degree, necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact, non-exact equations, rules for finding integrating factors (without proof) for non exact equations and examples

(4) Linear and reducible to linear equations, applications of first order ordinary differential equations.

Unit II: Second order Linear Differential equations (15 Lectures)

(1) Homogeneous and non-homogeneous second order linear differentiable equations, the space of solutions of the homogeneous equation as a vector space, wronskian and linear independence of the solutions, the general solution of homogeneous differential equation, the use of known solutions to find the general solution of homogeneous equations, the general solution of a non-homogeneous second order equation, complementary functions and particular integrals.

(2) The homogeneous equation with constant coefficient, auxiliary equation, the general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

(3) Non-homogeneous equations, the method of undetermined coefficients, the method of variation of parameters.

Unit III: Power Series solution of ordinary differential equations (15 Lectures)

1. A review of power series.
2. Power series solutions of first order ordinary differential equations.
3. Regular singular points of second order ordinary differential equations.
4. Frobenius series solution of second order ordinary differential equations with regular singular points.

Reference Books:

(1) G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.

(2) E. A. Coddington , An Introduction to Ordinary Differential Equations. Prentice Hall, 1961.

(3) W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiely, 2013.

(4) D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co., 1897.

(5) A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co., 1956.

Suggested Practicals for S.Y.B.Sc. and Tutorial for S.Y.B.A.:

1) Application of existence and uniqueness theorem, solving exact and non exact equations.
2) Linear and reducible to linear equations, applications to orthogonal trajectories, pop-ulation growth, and finding the current at a given time.
3) Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4) Solving equations using method of undetermined coefficients and method of variation of parameters.
5) Power series solutions of first order ordinary differential equations.
6) Frobenius series method for second order ordinary differential equatuions.

MODALITY OF ASSESSMENT

Theory Examination Pattern:

A) Internal Assessment - 40% :

(Except for RUSMAT303/RUAMAT302 and RUSMAT403/RUAMAT402)

Total: 40 marks.
1 One Assignment/Case study/Project/ seminars/presentation: 10 marks
2 One class Test (multiple choice questions / objective) 20 marks
3 Active participation in routine class instructional deliveries and Overall conduct as a responsible student, manners, skill in articulation, leadership qualities demonstrated through organizing co-curricular activities, etc. 10 marks

B) External examination - 60 %

Semester End Theory Assessment - 60 marks

i. Duration - These examinations shall be of 2 hours duration.

ii. Paper Pattern:

1. There shall be 3 questions each of 20 marks. On each unit there will be one question.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Options	Marks	Questions on
Q.1)A)	Any 1 out of 2	08	Unit I
Q.1)B)	Any 2 out of 4	12	
Q.2)A)	Any 1 out of 2	08	Unit II
Q.2)B)	Any 2 out of 4	12	
Q.3)A)	Any 1 out of 2	08	Unit III
Q.3)B)	Any 2 out of 4	12	

Practical Examination Pattern:

(For RUSMAT303/RUAMAT302 and RUSMAT403/RUAMAT402)

Journal 05 marks
Viva 05 marks
Test 30 marks
Total 40 marks

Res No: AB/II(18-19).2.RUA13 / AC/II(18-19).2.RUS8

S.P.Mandali's
RAMNARAIN RUIA AUTONOMOUS COLLEGE, MUMBAI-19

SYLLABUS FOR T.Y.B.Sc /T.Y.B.A

PROGRAM: B.Sc / B.A

COURSE: MATHEMATICS (RUSMAT/RUAMAT)

(Credit Based Semester and Grading System with effect from the academic year 2019–2020)

Semester V

Course Code	Unit	Topics	Credits	L/Week
Integral Calculus				
RUSMAT501/ RUAMAT501	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Algebra II				
RUSMAT502/ RUAMAT502	I	Group Theory	2.5	3
	II	Normal Subgroups		
	III	Direct Products of Groups		
Topology of Metric Spaces				
RUSMAT503/ RUAMAT503	I	Metric Spaces	2.5	3
	II	Closed Sets, Sequences and Completeness		
	III	Continuity		
Graph Theory (Elective I)				
RUSMATE504I/ RUAMATE504I	I	Basics of Graphs	2.5	3
	II	Trees		
	III	Eulerian and Hamiltonian graphs		
Number Theory and its Applications (Elective II)				
RUSMATE504II/ RUAMATE504II	I	Congruences and Factorization	2.5	3
	II	Diophantine Equations and their Solutions		
	III	Primitive Roots and Cryptography		
Course		**Practicals**	**Credits**	**L/Week**
RUSMATP501/ RUAMATP501		Practicals based on RUSMAT501/RUAMAT501 and RUSMAT502/RUAMAT502	3	6
RUSMATP502/ RUAMATP502		Practicals based on RUSMAT503/RUAMAT503, RUSMTE504I/RUAMTE504I or RUSMTE504II/RUAMTE504II	3	6

Semester VI

Course Code	Unit	Topics	Credits	L/Week
Basic Complex Analysis				
RUSMAT601/ RUAMAT601	I	Complex Numbers and Complex functions	2.5	3
	II	Holomorphic functions		
	III	Complex power series		
Algebra III				
RUSMAT602/ RUAMAT602	I	Ring Theory	2.5	3
	II	Factorization		
	III	Field Theory		
Metric Topology				
RUSMAT603/ RUAMAT603	I	Compact sets	2.5	3
	II	Connected sets		
	III	Sequences and Series of functions		
Graph Theory and Combinatorics (Elective I)				
RUSMAT604I/ RUSMATE604I	I	Colorings of graph	2.5	3
	II	Planar graph		
	III	Combinatorics		
Number Theory and its Applications II (Elective II)				
RUSMATE604II/ RUAMATE604II	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pells Equation, Arithmetic Functions, Special Numbers		
Course	**Practicals**		**Credits**	**L/Week**
RUSMATP601/ RUAMATP601	Practicals based on RUSMAT601/RUAMAT601 and RUSMAT602/RUAMAT602		3	6
RUSMATP602/ RUAMATP602	Practicals based on RUSMAT603/RUAMAT603, USMTE604I/UAMTE604I or USMTE604II/UAMTE604II		3	6

T.Y.B.Sc Mathematics
Semester V

Course: Integral Calculus

Course Code: RUSMAT501 / RUAMAT501

Learning Objectives:

1. Introduce notion of Multiple integrals.
2. To introduce notion of surface integrals, line integrals and their applications to Physics.

Learning Outcomes:

1. Learner will be able to apply concepts of multiple integrals in the field of physics.
2. Learner will be able to apply concepts of line integrals in the field of physics.
3. Learner will be able to apply concepts of surface integrals in the field of physics.

Detailed Syllabus

Unit I: Multiple Integrals (15 Lectures)

Definition of double (respectively: triple) integral of a function bounded on a rectangle (respectively: box), Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Basic properties of double and triple integrals proved using the Fubini's theorem such as; Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions, Formulae for the integrals of sums and scalar multiples of integrable functions, Integrability of continuous functions. More generally, integrability of bounded functions having finite number of points of discontinuity, Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only), Polar, cylindrical and spherical coordinates and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

Unit II: Line Integrals (15 Lectures)

Review of Scalar and Vector fields on $\mathbb{R}^n$. Vector Differential Operators, Gradient Paths (parametrized curves) in $\mathbb{R}$ (emphasis on $\mathbb{R}$ and $\mathbb{R}$), Smooth and piecewise smooth paths, Closed paths, Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path, Basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters, Examples.

Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative, Green's Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

Unit III: : Surface Integrals (15 Lectures)

Parameterized surfaces. Smoothly equivalent parameterizations, Area of such surfaces. Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface. Curl and divergence of a vector field, Elementary identities involving gradient, curl and divergence. Stoke's Theorem (proof assuming the general form of Green's Theorem), Examples. Gauss' Divergence Theorem (proof only in the case of cubical domains), Examples.

Reference Books:

(1) T. Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.

(2) R. Courant and F. John,, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.

(3) W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.

(4) M. H. Protter and C. B. Morrey, Jr., CIntermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.

(5) G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.

(6) D. V. Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989

(7) R Courant and F. John., Introduction to Calculus and Analysis, Vol I. Reprint of 1st Ed. Springer-Verlag, New York, 1999.

(8) Sudhir R. Ghorpade and Balmohan Limaye, A course in Multivariable Calculus and Analysis, Springer International Edition.

Course : Algebra II
Course Code : RUSMAT502/ RUAMAT502

Learning Objectives:

1. To introduce notion of Group and Subgroups
2. To introduce notion of direct product of groups.

Learning Outcomes:

1. Learner will be able to examine properties of groups and subgroups.
2. Learner will be able to identify normal subgroups.
3. Learner will be able to illustrate examples of direct product of groups.

Detailed Syllabus

Unit 1 : Group Theory

i. Groups, definition and properties, examples such as $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, GL_n(\mathbb{R})$, $SL_n(\mathbb{R})$, O_n (= the group of $n \times n$ real orthogonal matrices), B_n (= the group of $n \times n$ nonsingular upper triangular matrices), S_n, $\mathbb{Z}_n$, $U(n)$ the group of prime, residue classes modulo n under multiplication, Quarternion group, Dihedral group as group of symmetries of regular $n-$gon, abelian group, finite and infinite groups.

ii. Subgroups, necessary and sufficient condition for a non-empty subset of a group to be a subgroup. Examples, cyclic subgroups, centre $Z(G)$.

iii. Order of an element. Subgroup generated by a subset of the group. Cyclic group. Examples of cyclic groups such as $\mathbb{Z}$ and the group μ_n of the n–th roots of unity.

iv. Cosets of a subgroup in a group. Lagrange's Theorem.

v. Homomorphisms, isomorphisms, automorphisms, kernel and image of a homomorphism.

Unit 2 : Normal Subgroups

i. Normal subgroup of a group, centre of a group, Alternating group A_n, cycles, Quotient group.

ii. First Isomorphism Theorem, Second Isomorphism Theorem, Third Isomorphism Theorem, Correspondence Theorem.

iii. Permutation groups, cycle decomposition, Cayley's Theorem for finite groups..

iv. External direct product of groups, order of an element in a direct product, criterion for external product of finite cyclic groups to be cyclic.

v. Classification of groups of order ≤ 7

Unit 3 : Direct Product of Groups

i. Internal direct product of subgroups, H and K which are normal in G, such that $H \cap K = \{1\}$. If a group is internal direct product of two normal subgroups H and K and $HK = G$, it is isomorphic to the external direct product $H \times K$.

ii. Structure Theorem of finite abelian groups (statement only) and applications.

iii. Conjugacy classes in a group, class equation. A group of order p^2 is abelian.

Reference Books :

(1) I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.

(2) Michael Artin, Algebra, Prentice Hall of India, New Delhi.

(3) P.B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

(4) D. Dummit, R. Foote, Abstract Algebra, John Wiley and Sons, Inc.

Additional Reference Books :

(1) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

(2) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

(3) J. B. Fraleigh, A First Course in Abstract Algebra, Third edition, Narosa, New Delhi.

(4) T. W. Hungerford, Algebra, Springer.

Course: Topology of Metric Spaces

Course Code: RUSMAT503 /RUAMAT503

Learning Objectives:

1. To introduce notion of metric spaces, open sets closed sets in metric spaces
2. To introduce notion of continuity in metric spaces.

Learning Outcomes:

1. Learner will be able to construct examples of metrics.
2. Learner will be able to compare properties of open, closed intervals, sequences and completeness on R with an arbitrary metric space.
3. Learner will be able to compare properties of continuity on R with an arbitrary metric space.

Detailed Syllabus

Unit I: Metric Spaces (15 Lectures)

Definition, examples of metric spaces $\mathbb{R}$, $\mathbb{R}^2$ Euclidean space $\mathbb{R}^n$ sup and sum metric, $\mathbb{C}$ (complex numbers), normed spaces. distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces. Product of two metric spaces. Open balls and open sets in a metric space, examples of open sets in various metric spaces, Hausdorff property, interior of a set. Structure of an open set in $\mathbb{R}$, equivalent metrics. Distance of a point from a set, distance between sets, diameter of a set in a metric space and bounded sets.

Unit II: Closed sets, Sequences, Completeness (15 Lectures)

Closed ball in a metric space, Closed sets- definition, examples. Limit point of a set, Isolated point, A closed set contains all its limit points, Closure of a set and boundary, Sequences in a metric space, Convergent sequence in a metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces, $\mathbb{R}$ with different metrics and other metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces, Dense subsets in a metric space and Separability. Definition of complete metric spaces, Examples of complete metric spaces. Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's Intersection Theorem.

Unit III: Continuity (15 Lectures)

Epsilon-delta definition of continuity at a point of a function from one metric space to another. Equivalent characterizations of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space. Continuity of the composite of continuous functions.

Reference Books:

(1) S. Kumaresan, Topology of Metric spaces, Narosa, Second Edn.

(2) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

Additional Reference Books:

(1) W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

(2) T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

(3) P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.

(4) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

(5) D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis. Narosa, New Delhi

(6) G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.

(7) Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press, 2009

Course: Graph Theory (Elective I)

Course: Code RUSMATE504I / RUAMATE504I

Learning Objectives:

1. To introduce notion of Graph and its various attributes
2. To apply notion of graph to various branches of knowledge.

Learning Outcomes:

1. Learner will be able to apply the concepts of graphs and trees to the fields of chemistry, physics and biological sciences.
2. Learner will be able to apply the concepts of hamiltonian and eulerian to the fields of chemistry, physics and biological sciences.

Detailed Syllabus

Unit I: Basics of Graphs (15 Lectures)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem.

Unit II: Trees (15 Lectures)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of complete graphs, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs.

Unit III: Eulerian and Hamiltonian graphs (15 Lectures)

Eulerian graph and its characterization, Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G - S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs-Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of a graph and simple results.

Reference Books:

(1) Bondy and Murty, Graph Theory with Applications

(2) Balkrishnan and Ranganathan, Graph theory and applications.

(3) West D.B., Introduction to Graph Theory , Second ed., Prentice Hall 2001.

(4) Sharad Sane, Combinatorial Techniques, Hindustan Book Agency.

Additional Reference Books:

(1) Behzad and Chartrand , Graph theory

(2) Choudam S. A., Introductory Graph theory.

Course: Number Theory and its Applications (Elective II)
Course Code: RUSMATE504II / RUAMATE504II

Learning Objectives:

1. To introduce congruences and factorization
2. To introduce Dipophanine equations
3. To introduce Cryptography

Learning Outcomes:

1. Learner will be able to understand various aspects of factorization
2. Lerner will be able to understand importance of cryptography in todays world.

Detailed Syllabus

Unit 1 : Congruences and Factorization

Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m, Euler's function and its properties, Fermat's Little Theorem, Euler's generation of Fermat's Little Theorem, Wilson's Theorem, Linear congruence, The Chinese Remainder Theorem, Congruence of higher degree, The Fermat-Kraitchik Factorization Method.

Unit 2 : Diophantine Equations and their Solutions

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$ where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x, y, z) with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples –section 5.4 of Number theory by Niven-Zuckermann-Montgomery.

Unit 3 : Primitive Roots and Cryptography

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference Books :

(1) David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.

(2) Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley and Sons. Inc.

(3) M. Artin, Algebra. Prentice Hall.

(4) K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

Course: Practicals (Based on RUSMAT501 / RUAMAT501 and RUSMAT502 / RUAMAT502)

Course Code: RUSMATP501 / RUAMATP501

Suggested Practicals (Based on RUSMAT501 / RUAMAT501)

(1) Evaluation of double and triple integrals.
(2) Change of variables in double and triple integrals and applications.
(3) Line integrals of scalar and vector fields
(4) Green's theorem, conservative field and applications
(5) Evaluation of surface integrals
(6) Stoke's and Gauss divergence theorem
(()7) Miscellaneous theory questions.

Suggested Practicals (Based on RUSMAT502 / RUAMAT502)

(1) Examples and properties of groups
(2) Group of symmetry of equilateral triangle, rectangle, square.
(3) Subgroups
(4) Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
(5) Left and right cosets of a subgroup, Lagrange's Theorem.
(6) Group homomorphisms, isomorphisms.
(7) Miscellaneous theory questions.

Practicals
(Based on RUSMAT503/RUAMAT503, RUSMATE504I/RUAMATE504I and RUSMATE504II/RUAMATE504II)

Course Code: RUSMATP502/RUAMATP502

Suggested Practicals (Based on RUSMAT503/RUAMAT503)

(1) Examples of Metric Spaces.

(2) Open balls and Open sets in Metric / Normed Linear spaces, Interior Points.

(3) Subspaces, Closed Sets and Closure, Equivalent Metrics and Norms.

(4) Sequences, Convergent and Cauchy Sequences in a Metric Space, Complete Metric Spaces, Cantors Intersection Theorem and its Applications.

(5) Continuous Functions on Metric Spaces

(6) Characterization of continuity at a point in terms of metric spaces.

(7) Miscellaneous Theory Questions.

Suggested Practicals (Based on RUSMATE504I/RUAMATE504I)

(1) Handshaking Lemma and Isomorphism.

(2) Degree sequence.

(3) Trees, Cayley Formula.

(4) Applications of Trees.

(5) Eulerian Graphs.

(6) Hamiltonian Graphs.

(7) Miscellaneous Problems.

Suggested Practicals (Based on RUSMATE504II / RUAMATE504II)

(1) Congruences.

(2) Linear congruences and congruences of higher degree.

(3) Linear diophantine equations.

(4) Pythagorean triples and sum of squares.

(5) Cryptosystems (Private Key).

(6) Cryptosystems (Public Key) and primitive roots.

(7) Miscellaneous theoretical questions.

SEMESTER VI
Course: Basic Complex Analysis
Course Code: RUSMAT601 / RUAMAT601

Learning Objectives:

1. To introduce Complex Numbers , their subsets and complex-valued functions.
2. To introduce Mobius Transformations and singularities of sets of complex numbers.

Learning Outcomes:

1. Learner will be able to elaborate on properties of complex numbers.
2. Learner will be able to elaborate on properties of Mobius transforms and singularities in subsets of C.

Detailed Syllabus

Unit I: Complex Numbers and Functions of Complex variables (15 Lectures)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivr's formula, $\mathbb{C}$ as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane.

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions $f : \mathbb{C} \to \mathbb{C}$ real and imaginary part of functions, continuity at a point and algebra of continuous functions.

Unit II: Holomorphic functions (15 Lectures)

Derivative of $f : \mathbb{C} \to \mathbb{C}$; comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, $'$, f, g analytic then $f + g$, $f - g$, fg, f/g are analytic. chain rule.Theorem: If $f' = 0$ everywhere in a domain G then f must be constant throughout, Harmonic functions and harmonic conjugate.

Explain how to evaluate the line integral $\int f(z)dz$ over $|z - z_0| = r$ and prove the Cauchy integral formula: If f is analytic in $B(z_0, r)$ then for any w in $B(z_0, r)$ we have $f(w) = \displaystyle\int \frac{f(z)}{w - z} dz$ over $|z - z_0| = r$.

Unit III: Complex power series (15 Lectures)

Taylor's theorem for analytic functions, Mobius transformations –definition and examples. Exponential function, its properties, trigonometric function, hyperbolic functions, Power series of complex numbers and related results , radius of convergences, disc of convergence, uniqueness of series representation, examples.

Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, statement of residue theorem and calculation of residue.

Reference Books:

(1) J. W. Brown and R.V. Churchill, Complex analysis and Applications.

(2) S. Ponnusamy, Foundations Of Complex Analysis, Second Ed., Narosa, New Delhi. 1947

(3) R. E. Greene and S. G. Krantz, Function theory of one complex variable

(4) T. W. Gamelin, Complex analysis

Course: Algebra III
Course Code: RUSMAT602 / RUAMAT602

Semester VI
Course : Algebra III
Course Code : RUSMAT602/ RUAMAT602

Learning Objectives:

1. To introduce notion of Ring and ideal
2. To introduce factorization in commutative rings.
3. To introduce constructible numbers.

Learning Outcomes:

1. Learner will be able to extend concept of normal subgroup to ideal of the ring R.
2. Learner will be able to elaborate properties of ED, PID and UFD.
3. Learner will be able to find quadratic extensions of field F.

Detailed Syllabus

Unit 1 : Ring Theory

i. Ring (definition should include the existence of a unity element), zero divisor, unit, the multiplicative group of units of a ring. Basic properties and examples of rings.

ii. Commutative ring, integral domain, division ring, subring, examples, Characteristic of a ring, characteristic of an Integral Domain.

iii. Ring homomorphism, kernel of ring homomorphism, ideals, operations on ideals and quotient rings, examples.

iv. Factor theorem and First and Second isomorphism theorems for rings, Correspondence theorem for rings.

Unit 2 : Factorization

i. Principal ideal, maximal ideal, prime ideal, characterization of prime and maximal ideals in terms of quotient rings.

ii. Polynomial rings, R$[X]$ when R is an integral domain/ field, Eisenstein's criterion for irreducibility of a polynomial over $\mathbb{Z}$, Gauss lemma, prime and maximal ideals in polynomial rings.

iii Notions of euclidean domain (ED), principal ideal domain (PID) and unique factorization domain (UFD). Relation between these three notions (ED $\Rightarrow$ PID $\Rightarrow$ UFD).

iv Example of ring of Gaussian integers.

Unit 3 : Field Theory

i. Review of field, characteristic of a field, Characteristic of a finite field is prime.

ii. Prime subfield of a field, Prime subfield of any field is either $\mathbb{Q}$ or $\mathbb{Z}_p$ (upto isomorphism).

iii. Field extension, Degree of field extension. Algebraic elements, Any homomorphism of a field is injective.

iv. Any irreducible polynomial $p(x)$ over a field F has a root in an extension of the field, moreover the degree of this extension $\frac{F(x)}{(p(x))}$ over the field F is the degree of the polynomial $p(x)$.

v. The extension $\frac{\mathbb{Q}[x]}{(x^2-2)}$ *i.e.* $\mathbb{Q}(\sqrt{2})$, $\frac{\mathbb{Q}[x]}{(x^3-2)}$ *i.e.* $\mathbb{Q}(\sqrt[3]{2})$, $\frac{\mathbb{Q}[x]}{(x^2+1)}$ *i.e.* $\mathbb{Q}(i)$, Quadratic extensions of a field F when characteristic of F is not 2.

Reference Books :

(1) I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.

(2) Michael Artin, Algebra, Prentice Hall of India, New Delhi.

(3) P.B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

(4) D. Dummit, R. Foote, Abstract Algebra, John Wiley and Sons, Inc.

Additional Reference Books :

(1) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

(2) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

(3) J. B. Fraleigh, A First Course in Abstract Algebra, Third edition, Narosa, New Delhi.

(4) T. W. Hungerford, Algebra, Springer.

Course: Metric Topology

Course Code: RUSMAT603 /RUAMAT603

Learning Objectives:

1. To introduce notion of compactness and connectedness in Metric Spaces.
2. To introduce sequences and series of functions

Learning Outcomes:

1. Learner will be able to compare properties of compact and connected sets on R with an arbitrary metric spaces.
2. Learner will be able to elaborate on properties of sequences and series of functions.

Detailed Syllabus

Unit I: Compact Sets (15 Lectures)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$ and other metric spaces. Properties of compact sets:compact set is closed and bounded, every infinite bounded subset of a compact metric space has a limit point, Heine Borel theorem-every subset of Euclidean metric space $\mathbb{R}$ is compact if and only if it is closed and bounded. Equivalent statements for compact sets in $\mathbb{R}$; Heine-Borel property, Closed and boundedness property, Bolzano-Weierstrass property, Sequentially compactness property. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete.

Unit II: Connected sets (15 Lectures)

Separated sets- definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space. Connected subsets of $\mathbb{R}$, A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected, Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from b to $< 1, -1 >$ is a constant function. Path connectedness in $\mathbb{R}$, definition and examples, A path connected subset of $\mathbb{R}$ is connected, convex sets are path connected. Connected components, An example of a connected subset of $\mathbb{R}$ which is not path connected.

Unit III: Sequence and series of functions (15 Lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples. Consequences of these properties for series of functions, term by term differentiation and integration. Power series in $\mathbb{R}$ centered at origin and at some point $4F$ in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Reference Books:

(1) S. Kumaresan, Topology of Metric spaces. Narosa, Second Edn.

(2) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

(3) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

Additional Reference Books:

(1) W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

(2) T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

(3) E. T. Copson., Metric Spaces. Universal Book Stall, New Delhi, 1996.

(4) P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.

(5) D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis. Narosa, New Delhi

(6) G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.

(7) Sutherland, Introduction to Metric and Topological Spaces, Oxford University Press, 2009

Course: Graph Theory and Combinatorics (Elective I)
Course Code: RUSMATE604I /RUAMATE604I

Learning Objectives:

1. To introduce colorings of graphs and its applications in various fields of knowledge
2. To introduce a few combinatorial methods and its applications.

Learning Outcomes:

1. Learner will be able to apply the concepts of colorings of graphs and planar graph in the fields of chemistry, physics and biological sciences.
2. Learner will be able to apply the concepts of combinatorics in the field of statistics.

Detailed Syllabus

Unit I: Colorings of graphs (15 Lectures)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and Edge cuts vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II: Planar graphs (15 Lectures)

Definition of planar graph. Euler formula and its consequences. Non planarity of K_5; $K(3;3)$. Dual of a graph. Polyhedran in $\mathbb{R}$ and existence of exactly five regularpolyhedra- (Platonic solids) Colorabilty of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem.

Unit III: Combinatorics (15 Lectures)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems Introduction to partial fractions and using Newton's binomial theorem for real power series, expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR.

Reference Books:

(1) Bondy and Murty, Graph Theory with Applications

(2) Balkrishnan and Ranganathan, Graph theory and applications.

(3) West D G, Graph Theory

(4) Richard Brualdi, Introduction to Combinatorics.

(5) Sharad Sane, Combinatorial Techniques, Hindustan Book Agency.

Additional Reference Books:

(1) Behzad and Chartrand , Graph theory

(2) Choudam S. A., Introductory Graph theory.

(3) Cohen, Combinatorics

Course: Number Theory and its Applications II (Elective II)

Course Code: RUSMATE604II / RUAMATE604II

Learning Objectives:

1. To introduce quadratic reciprocity.
2. To introduce Continued Fractions.
3. To introduce spacial numbers in Number Theory

Learning Outcomes:

1. Learner will be able to apply Gauss Lemma in different situations.
2. Learner willbe able to undertsand continuied fractions.
3. Learner will be able to understand and apply theoy of arithmetic functions in simple situations.

Detailed Syllabus

Unit 1 : Quadratic Reciprocity

Quadratic Residues and Legendre Symbol, Euler's criterion, Gauss's Lemma, Quadratic Reciprocity Law. The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit 2 : Continued Fractions

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit 3 : Pell's Equation, Arithmetic Functions and Special Numbers

Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $T(n)$), $\sigma(n)$, $\sigma k(n)$, $w(n)$ and their properties, $\mu(n)$ and the Mobius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Reference Books :

(1) David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.

(2) Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley and Sons. Inc.

(3) M. Artin, Algebra. Prentice Hall.

(4) K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

Course: Practicals
(Based on RUSMAT601/RUAMT601 and RUSMAT602/RUAMAT602)

Course Code: RUSMATP601/RUAMATP601

Suggested Practicals (Based on RUSMAT601/RUAMAT601)

(1) Complex Numbers, subsets of $\mathbb{C}$ and their properties.

(2) Limits and continuity of complex-valued functions .

(3) Derivatives of functions of complex variables, analytic functions.

(4) Finding harmonic conjugate, Mobius transformations and Contour Integration.

(5) Cauchy integral formula, Taylor series, power series.

(6) Finding isolated singularities- removable, pole and essential, Laurent series, Calculation of residue.

(7) Miscellaneous theory questions.

Suggested Practicals (Based on RUSMAT602 / RUAMAT602)

(1) Rings, Subrings

(2) Ideals, Ring Homomorphism and Isomorphism

(3) Polynomial Rings

(4) Prime and Maximal Ideals

(5) Fields, Subfields

(6) Field Extensions

(7) Miscellaneous Theory Questions

Practicals
(Based on RUSMAT603/RUAMAT603 and RUSMATE604I/RUAMATE604I and RUSMATE604II/RUAMATE604II)

Course Code: RUSMATP602/RUAMATP602

Suggested Practicals (Based on RUSMAT603 / RUAMAT603)

(1) Examples of compact metric spaces.

(2) Equivalent conditions for a subset of a metric space to be compact

(3) Connectedness

(4) Path Connectedness

(5) Pointwise and uniform convergence of sequence and series of functions and their properties.

(6) Power series and Elementary functions.

(7) Miscellaneous Theory Questions.

Suggested Practicals (Based on RUSMATE604I / RUAMATE604I)

(1) Coloring of Graphs.

(2) Chromatic polynomials and connectivity.

(3) Planar graphs.

(4) Generating Functions.

(5) Rook polynomial.

(6) System of Distinct Representatives.

(7) Miscellaneous Problems.

Suggested Practicals (Based on RUSMATE604II / RUAMATE604II)

(1) Legendre Symbol.

(2) Jacobi Symbol and Quadratic congruences with composite moduli.

(3) Finite continued fractions.

(4) Infinite continued fractions.

(5) Pell's equations and Arithmetic functions of number theory.

(6) Special Numbers.

(7) Miscellaneous theoretical questions.

MODALITY OF ASSESSMENT

Theory Examination Pattern:
A) Internal Assessment - 40% :

Total: 40 marks.
1 One Assignment/Case study/Project/ seminars/presentation: 10 marks
2 One class Test (multiple choice questions / objective) 20 marks
3 Active participation in routine class instructional deliveries and Overall conduct as a responsible student, manners, skill in articulation, leadership qualities demonstrated through organizing co-curricular activities, etc. 10 marks

B) External examination - 60 %
Semester End Theory Assessment - 60 marks

i. Duration - These examinations shall be of 2 hours duration.

ii. Paper Pattern:

1. There shall be 3 questions each of 20 marks. On each unit there will be one question.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Options	Marks	Questions on
Q.1)A)	Any 1 out of 2	08	Unit I
Q.1)B)	Any 2 out of 4	12	
Q.2)A)	Any 1 out of 2	08	Unit II
Q.2)B)	Any 2 out of 4	12	
Q.3)A)	Any 1 out of 2	08	Unit III
Q.3)B)	Any 2 out of 4	12	

Practical Examination Pattern:
(A)Internal Examination:

Journal 05 marks
Test 15 marks
Total 20

(B) External (Semester end practical examination):
Particulars:
Test: 30 marks

PRACTICAL BOOK/JOURNAL

- The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.
- In case of loss of Journal and/ or Report, a Lost Certificate should be obtained from Head/ Co-ordinator / Incharge of the department; failing which the student will not be allowed to appear for the practical examination.

****